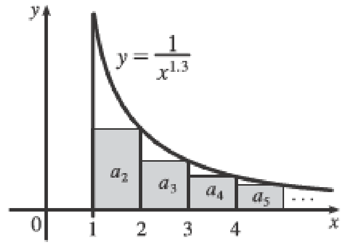
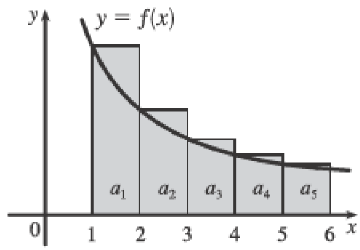
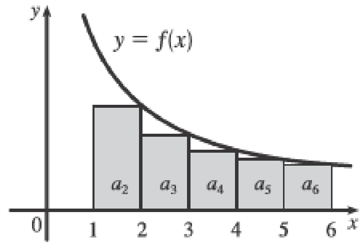
1. The picture shows that $a_2 = \dfrac{1}{2^{1.3}} < \displaystyle\int_1^2 \frac{1}{x^{1.3}}\,dx$,

$a_3 = \dfrac{1}{3^{1.3}} < \displaystyle\int_2^3 \frac{1}{x^{1.3}}\,dx$, and so on, so $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^{1.3}} < \int_1^{\infty} \frac{1}{x^{1.3}}\,dx$. The integral converges by (7.8.2) with $p = 1.3 > 1$, so the series converges.



2. From the first figure, we see that $\int_1^6 f(x)\,dx < \sum_{i=1}^{5} a_i$. From the second figure, we see that $\sum_{i=2}^{6} a_i < \int_1^6 f(x)\,dx$. Thus, we have $\sum_{i=2}^{6} a_i < \int_1^6 f(x)\,dx < \sum_{i=1}^{5} a_i$.





3. The function $f(x) = 1/\sqrt[5]{x} = x^{-1/5}$ is continuous, positive, and decreasing on $[1, \infty)$, so the Integral Test applies.

$\int_1^{\infty} x^{-1/5}\,dx = \lim\limits_{t\to\infty} \int_1^t x^{-1/5}\,dx = \lim\limits_{t\to\infty} \left[\frac{5}{4}x^{4/5}\right]_1^t = \lim\limits_{t\to\infty} \left(\frac{5}{4}t^{4/5} - \frac{5}{4}\right) = \infty$, so $\sum\limits_{n=1}^{\infty} 1/\sqrt[5]{n}$ diverges.

7. $f(x) = xe^{-x}$ is continuous and positive on $[1, \infty)$. $f'(x) = -xe^{-x} + e^{-x} = e^{-x}(1 - x) < 0$ for $x > 1$, so f is decreasing on $[1, \infty)$. Thus, the Integral Test applies.

$$\int_1^{\infty} xe^{-x}\,dx = \lim_{b\to\infty} \int_1^b xe^{-x}\,dx = \lim_{b\to\infty} \left[-xe^{-x} - e^{-x}\right]_1^b \quad \text{[by parts]} = \lim_{b\to\infty} [-be^{-b} - e^{-b} + e^{-1} + e^{-1}] = 2/e$$

since $\lim\limits_{b\to\infty} be^{-b} = \lim\limits_{b\to\infty} (b/e^b) \overset{\text{H}}{=} \lim\limits_{b\to\infty} (1/e^b) = 0$ and $\lim\limits_{b\to\infty} e^{-b} = 0$. Thus, $\sum_{n=1}^{\infty} ne^{-n}$ converges.

8. The function $f(x) = \dfrac{x+2}{x+1} = 1 + \dfrac{1}{x+1}$ is continuous, positive, and decreasing on $[1, \infty)$, so the Integral Test applies.

$$\int_1^{\infty} f(x)\,dx = \lim_{t\to\infty} \int_1^t \left(1 + \frac{1}{x+1}\right) dx = \lim_{t\to\infty} \left[x + \ln(x+1)\right]_1^t = \lim_{t\to\infty} (t + \ln(t+1) - 1 - \ln 2) = \infty, \text{ so}$$

$\displaystyle\int_1^{\infty} \frac{x+2}{x+1}\,dx$ is divergent and the series $\displaystyle\sum_{n=1}^{\infty} \frac{n+2}{n+1}$ is divergent.

Note: $\displaystyle\lim_{n\to\infty} \frac{n+2}{n+1} = 1$, so the given series diverges by the Test for Divergence.

9. The series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{0.85}}$ is a p-series with $p = 0.85 \leq 1$, so it diverges by (1). Therefore, the series $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n^{0.85}}$ must also diverge, for if it converged, then $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{0.85}}$ would have to converge [by Theorem 8(i) in Section 11.2].

10. $\sum_{n=1}^{\infty} n^{-1.4}$ and $\sum_{n=1}^{\infty} n^{-1.2}$ are p-series with $p > 1$, so they converge by (1). Thus, $\sum_{n=1}^{\infty} 3n^{-1.2}$ converges by Theorem 8(i) in Section 11.2. It follows from Theorem 8(ii) that the given series $\sum_{n=1}^{\infty} (n^{-1.4} + 3n^{-1.2})$ also converges.

11. $1 + \frac{1}{8} + \frac{1}{27} + \frac{1}{64} + \frac{1}{125} + \cdots = \sum_{n=1}^{\infty} \frac{1}{n^3}$. This is a p-series with $p = 3 > 1$, so it converges by (1).

12. $1 + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} + \frac{1}{5\sqrt{5}} + \cdots = \sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$. This is a p-series with $p = \frac{3}{2} > 1$, so it converges by (1).

15. $\sum_{n=1}^{\infty} \frac{5 - 2\sqrt{n}}{n^3} = 5 \sum_{n=1}^{\infty} \frac{1}{n^3} - 2 \sum_{n=1}^{\infty} \frac{1}{n^{5/2}}$ by Theorem 11.2.8, since $\sum_{n=1}^{\infty} \frac{1}{n^3}$ and $\sum_{n=1}^{\infty} \frac{1}{n^{5/2}}$ both converge by (1) $\left[\text{with } p = 3 > 1 \text{ and } p = \frac{5}{2} > 1\right]$. Thus, $\sum_{n=1}^{\infty} \frac{5 - 2\sqrt{n}}{n^3}$ converges.

20. The function $f(x) = \frac{1}{x^2 - 4x + 5} = \frac{1}{(x-2)^2 + 1}$ is continuous, positive, and decreasing on $[2, \infty)$, so the Integral Test applies.

$$\int_2^{\infty} f(x)\,dx = \lim_{t\to\infty} \int_2^t f(x)\,dx = \lim_{t\to\infty} \int_2^t \frac{1}{(x-2)^2+1}\,dx = \lim_{t\to\infty} \left[\tan^{-1}(x-2)\right]_2^t = \lim_{t\to\infty} [\tan^{-1}(t-2) - \tan^{-1} 0]$$
$$= \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

so the series $\sum_{n=2}^{\infty} \frac{1}{n^2 - 4n + 5}$ converges. Of course, this means that $\sum_{n=1}^{\infty} \frac{1}{n^2 - 4n + 5}$ converges too.

30. If $p \leq 0$, $\lim_{n\to\infty} \frac{\ln n}{n^p} = \infty$ and the series diverges, so assume $p > 0$. $f(x) = \frac{\ln x}{x^p}$ is positive and continuous and $f'(x) < 0$ for $x > e^{1/p}$, so f is eventually decreasing and we can use the Integral Test. Integration by parts gives

$$\int_1^{\infty} \frac{\ln x}{x^p} dx = \lim_{t\to\infty} \left[\frac{x^{1-p}\,[(1-p)\ln x - 1]}{(1-p)^2}\right]_1^t \quad (\text{for } p \neq 1) = \frac{1}{(1-p)^2} \left[\lim_{t\to\infty} t^{1-p}\,[(1-p)\ln t - 1] + 1\right],$$ which exists

whenever $1 - p < 0 \iff p > 1$. Thus, $\sum_{n=1}^{\infty} \frac{\ln n}{n^p}$ converges $\iff p > 1$.

32. (a) $f(x) = 1/x^4$ is positive and continuous and $f'(x) = -4/x^5$ is negative for $x > 0$, and so the Integral Test applies.

$$\sum_{n=1}^{\infty} \frac{1}{n^4} \approx s_{10} = \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots + \frac{1}{10^4} \approx 1.082037.$$

$R_{10} \le \int_{10}^{\infty} \frac{1}{x^4}\,dx = \lim_{t\to\infty} \left[\frac{1}{-3x^3}\right]_{10}^{t} = \lim_{t\to\infty}\left(-\frac{1}{3t^3} + \frac{1}{3\,(10)^3}\right) = \frac{1}{3000}$, so the error is at most $0.000\overline{3}$.

(b) $s_{10} + \int_{11}^{\infty} \frac{1}{x^4}\,dx \le s \le s_{10} + \int_{10}^{\infty} \frac{1}{x^4}\,dx \quad\Rightarrow\quad s_{10} + \frac{1}{3(11)^3} \le s \le s_{10} + \frac{1}{3(10)^3} \quad\Rightarrow$

$1.082037 + 0.000250 = 1.082287 \le s \le 1.082037 + 0.000333 = 1.082370$, so we get $s \approx 1.08233$ with error ≤ 0.00005.

(c) $R_n \le \int_{n}^{\infty} \frac{1}{x^4}\,dx = \frac{1}{3n^3}$. So $R_n < 0.00001 \quad\Rightarrow\quad \frac{1}{3n^3} < \frac{1}{10^5} \quad\Rightarrow\quad 3n^3 > 10^5 \quad\Rightarrow\quad n > \sqrt[3]{(10)^5/3} \approx 32.2$, that is, for $n > 32$.

35. $f(x) = 1/(2x+1)^6$ is continuous, positive, and decreasing on $[1, \infty)$, so the Integral Test applies. Using (2),

$R_n \le \int_{n}^{\infty} (2x+1)^{-6}\,dx = \lim_{t\to\infty} \left[\frac{-1}{10(2x+1)^5}\right]_{n}^{t} = \frac{1}{10(2n+1)^5}$. To be correct to five decimal places, we want

$\frac{1}{10(2n+1)^5} \le \frac{5}{10^6} \quad\Leftrightarrow\quad (2n+1)^5 \ge 20{,}000 \quad\Leftrightarrow\quad n \ge \frac{1}{2}\left(\sqrt[5]{20{,}000} - 1\right) \approx 3.12$, so use $n = 4$.

$$s_4 = \sum_{n=1}^{4} \frac{1}{(2n+1)^6} = \frac{1}{3^6} + \frac{1}{5^6} + \frac{1}{7^6} + \frac{1}{9^6} \approx 0.001\,446 \approx 0.00145.$$