MA 202 § 12.2 #s 1-3, 7-12, 19, 20, 22, 25
29-31, 35, 36, 41, 42, 47, 48, 52, 65,
69, 70

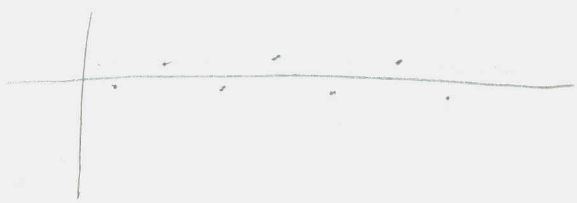
(1a) The difference between sequence and series is that a series SUMS all the terms.

(1b) A convergent series $S = \sum_{k=1}^{\infty} a_k$ is one for which the sequence $\{S_n\}$ of partial sums $S_n = \sum_{k=1}^{n} a_k$ is convergent. Otherwise, S is divergent (NOT CONVERGENT)

(2) To say that $\sum_{k=1}^{\infty} a_k = 5$ is to say that the sequence of n^{th} partial sums converges to 5.

#s 3-8 Find at least 10 partial sums Graph both the sequence of terms & the sequence of partial sums.

(3) $\sum_{k=1}^{\infty} \frac{12}{(-5)^k} = \sum_{k=1}^{\infty} 12\left(-\frac{1}{5}\right)^k$



a_k	S_k
-2.4	-2.4
.48	-1.92
-.096	-2.016
+.0192	-1.997
-.0038	-2.001
7.7×10^{-4}	-2
	⋮

(202) §12.2 #s 9-12, 16, 19, 20, 22, 25, 29-31
35, 36, 41, 42, 47, 48, 52, 65, 69, 70

(9) $a_n \equiv \frac{2n}{3n+1}$

(a) $\{a_n\}$ converges to $\frac{2}{3}$

(b) $\sum a_n$ diverges, because $a_n \not\to 0$, and that's necessary.

(10) (a) There's <u>no</u> essential difference between $\sum_{i=1}^{n} a_i$ and $\sum_{j=1}^{n} a_j$. Just a different index.

(b) $\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_n$, but

$\sum_{i=1}^{n} a_j = a_j + a_j + \cdots + a_j = n\, a_j$

Index i doesn't match subscript j.

#s 11-20 Does the geometric series converge? If so, then to what?

(11) $3 + 2 + \frac{4}{3} + \frac{8}{9} + \cdots$

$r = \frac{2}{3}$. It converges.

$S = \frac{a}{1-r} = \frac{3}{1-\frac{2}{3}}$

$a = a_1 = 3 = 3\cdot\left(\frac{2}{3}\right)^0$

$= \frac{3}{\frac{1}{3}} = \boxed{9 = S}$

$2 = a_2 = 3\left(\frac{2}{3}\right)^1$

202 §12.2 #s 12, 16, 19, 20, 22, 25, 29-31, 35, 36, 41, 42, 47, 48, 52, 65, 69, 70

(12) $\frac{1}{8} - \frac{1}{4} + \frac{1}{2} - 1 + \cdots$

$r = -2 \qquad a = \frac{1}{8} = a_1$

$a_2 = -\frac{1}{4} = \left(\frac{1}{8}\right)(-2)$

Because $|r| > 1$, this series diverges.

(16) $\sum_{n=1}^{\infty} \frac{10^n}{(-9)^{n-1}} = \sum_{n=1}^{\infty} \frac{10 \cdot 10^{n-1}}{(-9)^{n-1}} = \sum_{n=1}^{\infty} 10 \cdot \left(-\frac{10}{9}\right)^{n-1}$

is divergent

(19) $\sum_{n=0}^{\infty} \frac{\pi^n}{3^{n+1}} = \frac{1}{3} + \frac{\pi}{3^2} + \frac{\pi^2}{3^3} + \cdots + \left(\frac{1}{3}\right)\frac{\pi^n}{3^n} + \cdots$

$a_1 = a = \frac{1}{3}$, $\boxed{r = \frac{\pi}{3}}$ Re-write: $\sum_{n=1}^{\infty} \frac{1}{3} \cdot \left(\frac{\pi}{3}\right)^{n-1}$

$\boxed{\text{Divergent}}$

(20) $\sum_{n=1}^{\infty} \frac{e^n}{3^{n-1}} = \sum_{n=1}^{\infty} e \cdot \frac{e^{n-1}}{3^{n-1}} \qquad r = \frac{e}{3}, \; a = e$

Converges to $a\left(\frac{1}{1-r}\right) = e\left(\frac{1}{1-\frac{e}{3}}\right) = e\left(\frac{1}{\frac{3-e}{3}}\right)$

$= \boxed{\frac{3e}{3-e} = S}$

#s 21-34 Converges? If so, find its sum.

(22) $\sum_{n=1}^{\infty} \frac{n+1}{2n-3}$ Diverges $\quad a_n \not\longrightarrow 0$

$\left(a_n \xrightarrow{n \to \infty} \frac{1}{2}\right)$

202 §12.2 #s 25, 29–31, 35, 36, 41, 42, 47, 48, 52, 65, 69, 70

(25) $\sum_{n=1}^{\infty} \frac{1+2^n}{3^n} = \sum_{n=1}^{\infty}\left(\frac{1}{3^n} + \frac{2^n}{3^n}\right)$ converges,

since $\sum_{n=1}^{\infty} \frac{1}{3^n}$ & $\sum\left(\frac{2}{3}\right)^n$ converge.

$\sum_{n=1}^{\infty} \frac{1}{3}\left(\frac{1}{3}\right)^{n-1} = \frac{1}{3}\left(\frac{1}{1-\frac{1}{3}}\right) = \frac{1}{3}\left(\frac{1}{\frac{2}{3}}\right) = \left(\frac{1}{3}\right)\left(\frac{3}{2}\right) = \frac{1}{2}$

$\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n = \sum_{n=1}^{\infty}\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)^{n-1} = \frac{2}{3}\left(\frac{1}{1-\frac{2}{3}}\right) = \frac{2}{3}\cdot\left(\frac{1}{\frac{1}{3}}\right) = \left(\frac{2}{3}\right)(3)$

$= 2$, so, the original sum is

$\frac{1}{2} + 2 = \boxed{\frac{5}{2} = 5}$

(29) $\sum_{n=1}^{\infty} \ln\left(\frac{n^2+1}{2n^2+1}\right)$ is $\boxed{\text{divergent}}$

$a_n = \ln\left(\frac{n^2+1}{2n^2+1}\right) \xrightarrow{n\to\infty} \ln\left(\frac{1}{2}\right) \neq 0$

(30) $\sum_{k=1}^{\infty} (\cos 1)^k = \sum_{k=1}^{\infty} (\cos 1)(\cos 1)^{k-1}$

$= (\cos 1)\left(\frac{1}{1-\cos 1}\right) = \boxed{\frac{\cos 1}{1-\cos 1}} \approx \frac{.540323059}{.4596976941}$

$\approx \boxed{1.17534265}$

202 §12.2 #s 31, 35, 36, 41, 42, 47, 48, 52, 65, 69, 70

31 $\sum_{n=1}^{\infty} \arctan n$ Divergent

$\arctan(n) \xrightarrow{n\to\infty} \frac{\pi}{2} \neq 0$

#s 35 – 40 Determine if it's convergent by writing s_n as a telescoping sum. If convergent, find its sum

35 $\sum_{n=2}^{\infty} \frac{2}{n^2-1}$ $\frac{2}{n^2-1} = \frac{A}{n-1} + \frac{B}{n+1}$

$2 = A(n+1) + B(n-1)$

$An + Bn = 0 \longrightarrow A = -B$

$A - B = 2$

$-B - B = 2$

$-2B = 2$

$\boxed{B = -1} \longrightarrow \boxed{A = 1}$

$$\sum_{n=2}^{\infty} \frac{2}{n^2-1} = \sum_{n=2}^{\infty}\left[\frac{1}{n-1} - \frac{1}{n+1}\right] = \frac{1}{1} - \frac{1}{3} + \frac{1}{2} - \frac{1}{4}$$

$$+ \frac{1}{3} - \frac{1}{5} + \frac{1}{4} - \frac{1}{6} + \frac{1}{5} - \frac{1}{7} + \frac{1}{6} - \frac{1}{8} + \cdots + \frac{1}{n-1} - \frac{1}{n+1}$$

$$= \frac{1}{1} + \frac{1}{2} - \frac{1}{n} - \frac{1}{n+1} = \frac{3}{2} - \frac{1}{n} - \frac{1}{n+1} \xrightarrow{n\to\infty} \boxed{\frac{3}{2}}$$

202 $\S$ 12.2 #s 36, 41, 42, 47, 48, 52, 65, 69, 70

36 $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n^2+4n+3}$ $\qquad \displaystyle\frac{2}{n^2+4n+3} = \frac{A}{n+1} + \frac{B}{n+3}$

$2 = A(n+3) + B(n+1)$ $\qquad \displaystyle = \frac{1}{n+1} - \frac{1}{n+3}$

$$\boxed{\begin{array}{l} 3A + B = 2 \\ A + B = 0 \longrightarrow A = -B \end{array}}$$

So $-3B + B = 2$

$-2B = 2$

$\boxed{B = -1 \longrightarrow A = 1}$

$$\therefore \sum_{m=1}^{n} \frac{2}{m^2+4m+3} = \sum_{m=1}^{n} \left[\frac{1}{m+1} - \frac{1}{m+3}\right]$$

$$= \frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \frac{1}{5} + \frac{1}{4} - \frac{1}{6} + \frac{1}{5} - \frac{1}{7} + \frac{1}{6} - \frac{1}{8}$$

$$+ \dots + \frac{1}{n+1} - \frac{1}{n+3} = \frac{1}{2} + \frac{1}{3} - \frac{1}{n+1} - \frac{1}{n+3}$$

$$\xrightarrow{n \to \infty} \boxed{\frac{5}{6}}$$

41 $.\overline{2} = .222222\ldots = x$

$2.222222\ldots = 10x$

$2 = 9x$

$\boxed{\frac{2}{9} = x}$

202 §12.2 #s 42, 47, 48, 52, 65, 69, 70

(42) $.\overline{73} = .737373\ldots = x$

$73.737373\ldots = 100x$

$73 = 99x$

$\boxed{\frac{73}{99} = x}$

#s 47-51 Find $x \ni$ series converges. Find sum if convergent.

(47) $\sum_{n=1}^{\infty} \frac{x^n}{3^n} = \sum_{n=1}^{\infty} \left(\frac{x}{3}\right)\left(\frac{x}{3}\right)^{n-1}$ Geometric

Need $\frac{x}{3} < 1$

Need $\left|\frac{x}{3}\right| < 1 \longrightarrow$

$-1 < \frac{x}{3} < 1 \longrightarrow$

$\boxed{-3 < x < 3 \text{ is restriction on } x}$

$\frac{x}{3}\left(\frac{1}{1-\frac{x}{3}}\right)$

$= \frac{x}{3}\left(\frac{1}{\frac{3-x}{3}}\right)$

$= \frac{3x}{3(3-x)} = \boxed{\frac{x}{3-x}}$

(48) $\sum_{n=1}^{\infty} (x-4)^n = \sum_{n=1}^{\infty} (x-4)(x-4)^{n-1}$

Converges if $|x-4| < 1 \longrightarrow$

$-1 < x-4 < 1 \longrightarrow$

$\boxed{3 < x < 5}$

$(x-4)\left(\frac{1}{1-(x-4)}\right)$

$= (x-4)\left(\frac{1}{5-x}\right)$

$= \boxed{\frac{x-4}{5-x}}$

202 § 12.2 #s 52, 65, 69, 70

52 Show $\sum_{n=1}^{\infty} \ln(1+\frac{1}{n})$ diverges, even though

$\ln(1+\frac{1}{n}) \xrightarrow{n\to\infty} 0$. To see this,

$\ln(1+\frac{1}{n}) = \ln(\frac{n+1}{n}) = \ln(n+1) - \ln(n)$,

so $\sum_{k=1}^{n} \ln(1+\frac{1}{k}) = \ln(2) - \ln 1 + \ln 3 - \ln 2$

$+ \ln 4 - \ln 3 + \ln 5 - \ln 4 + \cdots + \ln(n+1) - \ln(n)$

$= \ln(n+1) - \ln 1 \xrightarrow{n\to\infty} \infty$ ▨

65 $0 = 0 + 0 + \cdots + 0 + \cdots$

$= (1-1) + (1-1) + \cdots + (1-1) + \cdots$

$= 1 + (-1+1) + (-1+1) + \cdots (-1+1) + \cdots$

$= 1$ The problem with this is

that the geometric series $1 - 1 + 1 - 1 \ldots$

diverges, so saying it equals zero is

not appropriate.

69 If $\sum a_n$ converges & $\sum b_n$ diverges

then $\sum (a_n + b_n)$ diverges.

$\boxed{\text{Pf}}$ S $\sum(a_n+b_n)$ converges, $\sum a_n$ converges

& $\sum b_n$ diverges.

202 $\S$ 12.2 #s 69, 70

69 Then, by properties of Series,

we have that $\sum(a_n+b_n-a_n)$ converges.

But this means that $\sum b_n$ converges!?

$\times$ $\therefore$ $\sum(a_n+b_n)$ diverges $\blacksquare$

70 If $\sum a_n$ & $\sum b_n$ are divergent, is

$\sum(a_n+b_n)$ divergent?

No. $a_n=1$, $b_n=-1$ $\longrightarrow$

$\sum(a_n+b_n)=\sum 0=0$ is convergent,

But $\sum 1$ & $\sum(-1)$ are divergent.