202 §12.10 II #s 39, 44, 47, 51, 54, 55, 56, 63, 66

(39) As $n \to$ bigger, $T_n \to$ better

(44) Use Maclaurin series for $\sin x$ to compute $\sin(3^\circ)$ correct to 5 decimal places

$$\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$\sin(3^\circ) = \sin\left(3^\circ \cdot \frac{\pi}{180^\circ}\right) = \sin\left(\frac{\pi}{60}\right)$$

$$: \frac{\pi}{60} - \frac{\left(\frac{\pi}{60}\right)^3}{3!} + \frac{\left(\frac{\pi}{60}\right)^5}{5!} + \cdots, \text{ but } \frac{\left(\frac{\pi}{60}\right)^5}{5!} < 10^{-8}$$

so, using 1st 2 terms: $\boxed{\sin(3^\circ) \approx .05234}$

cool! Just 2 terms needed!

(47) Evaluate using series

$$\int x \cos(x^3)\,dx = \int x \sum_{n=0}^{\infty} (-1)^n \frac{(x^3)^{2n}}{(2n)!}\,dx$$

$$= \int \sum_{n=0}^{\infty} (-1)^n \frac{x^{6n+1}}{(2n)!}\,dx = \boxed{\sum_{n=0}^{\infty} (-1)^n \frac{x^{6n+2}}{(6n+2)(2n)!} \quad R = \infty}$$

(51) #s 51-54 Find approximation to the integral with power series. Meet the desired accuracy.

202 §12.10 III #551, 54, 55, 56

51 $\displaystyle\int_0^1 x\cos(x^3)\,dx$ 3 decimal places

By #47, $\displaystyle\left.\sum_{n=0}^{\infty}(-1)^n\frac{x^{6n+2}}{(6n+2)(2n)!}\right]_{x=0}^{x=1}$

$$= \frac{1}{2} - \frac{1}{8(2)!} + \frac{1}{14\cdot 4!} - \frac{1}{20\cdot 6!} + \cdots$$

$\approx \frac{1}{2} - \frac{1}{16} + \frac{1}{336} \approx \boxed{.440}$, since $\frac{1}{20\cdot 6!} \approx .000069$

I still think you should go & look @ the next sum, to make sure the 3rd digit changes. I'm thinking about something like .4404999 which an .000069 would bump to .441, if you bump a 5 up.

54 #56 $\displaystyle\int_0^{.5} x^2 e^{-x^2}\,dx$ (|error| < .001)

$$f(x) = x^2\sum_{n=0}^{\infty}(-1)^n\frac{(x^2)^n}{n!} = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+2}}{n!} \Rightarrow$$

$$\int_0^{.5} f(x)\,dx = \left.\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+3}}{(2n+3)n!}\right]_0^{.5} \approx \boxed{a_2 = \frac{1}{1792} < .001}$$

$$\frac{.5^3}{3\cdot 0!} - \frac{.5^5}{5\cdot 1} \approx \frac{1}{24} - \frac{1}{160} \approx \boxed{.0354}$$

202 S'12.10 II #s 55, 56

#s 55–57 Use series to evaluate the limit.

55 $\lim_{x \to 0} \frac{x - \arctan(x)}{x^3}$

$= \lim_{x \to 0} \left(\frac{1}{x^3} \left(x - \arctan(x) \right) \right) =$

$= \lim_{x \to 0} \left(\frac{1}{x^3} \left(x - \left(x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \cdots \right) \right) \right)$

$= \lim_{x \to 0} \left(\frac{1}{x^3} \left(\frac{1}{3}x^3 - \frac{1}{5}x^5 + \frac{1}{7}x^7 + \cdots \right) \right)$

$= \lim_{x \to 0} \left(\frac{1}{3} - \frac{1}{5}x^2 + \frac{1}{7}x^4 + \cdots \right) = \boxed{\frac{1}{3}}$

56 $\lim_{x \to 0} \frac{1 - \cos(x)}{1 + x - e^x} = \lim_{x \to 0} \frac{1 - \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}}{1 + x - 1 - x - \frac{x^2}{2} - \frac{x^3}{3!} - \cdots}$

$= \lim_{x \to 0} \frac{1 - 1 + \frac{x^2}{2} - \frac{x^4}{4!} + \frac{x^6}{6!} - \cdots}{-\frac{x^2}{2} - \frac{x^3}{3} - \frac{x^5}{5} - \cdots}$

$= \lim_{x \to 0} \frac{\frac{x^2}{2} - \frac{x^4}{4} + \frac{x^6}{6} + \cdots}{-\frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} + \cdots} = \lim_{x \to 0} \frac{\frac{1}{2} - \frac{x^2}{4} + \frac{x^4}{6} + \cdots}{-\frac{1}{2} - \frac{x}{3} - \frac{x^2}{4} - \cdots}$

$= \boxed{-1}$ Heh!