202 §12.8 #s 1, 2, 4, 7, 10, 13, 16, 19, 22, 25, 28, 32, 33

#s 2-28 Find radius & interval of convergence.

3 $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$

$\left| \frac{x^{n+1}}{\sqrt{n+1}} \cdot \frac{\sqrt{n}}{x^n} \right| = \left| \frac{\sqrt{n}}{\sqrt{n+1}} \right| |x| \xrightarrow{n \to \infty} |x| \overset{\text{want}}{<} 1$ so

$\boxed{R = 1}$.

Now, $x = 1$: $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ diverges

$x = -1$: $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ converges

$\boxed{IC = [-1, 1)}$

4 $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n+1}$

$\left| \frac{x^{n+1}}{n+2} \cdot \frac{n+1}{x^n} \right| = \left| \frac{n+1}{n+2} \right| |x| \xrightarrow{n \to \infty} |x| \overset{\text{want}}{<} 1$

$\boxed{R = 1}$

$x = 1$: $\sum_{n=0}^{\infty} \frac{(-1)^n (1)^n}{n+1}$ converges

#10

$x = -1$ $\sum_{n=0}^{\infty} \frac{(-1)^n (-1)^n}{n+1} = \sum_{n=0}^{\infty} \frac{1}{n+1}$ diverges

$\boxed{IC = (-1, 1]}$

202 §12.8 #s 7, 10, 13, 16, 19, 22, 25, 28, 32, 33

(7) $\sum_{n=0}^{\infty} \frac{x^n}{n!}$

$$\left| \frac{x^{n+1}}{(n+1)!} \cdot \frac{n!}{x^n} \right| = \frac{|x|}{n+1} \xrightarrow{n \to \infty} 0 \to$$

$$\boxed{R = \infty \ \& \ IC = (-\infty, \infty)}$$

(10) $\sum_{n=1}^{\infty} \frac{10^n x^n}{n^3}$

~~#16~~

$$\left| \frac{10^{n+1} x^{n+1}}{(n+1)^3} \cdot \frac{n^3}{10^n x^n} \right| = \left| \frac{n^3}{(n+1)^3} \right| |10x| \xrightarrow{n \to \infty} |10x| \ \text{want} < 1$$

$$\Rightarrow |x| < \boxed{\frac{1}{10} = R}$$

$x = \frac{1}{10} \Rightarrow \sum_{n=0}^{\infty} \frac{10^n (\frac{1}{10})^n}{n^3} = \sum_{n=0}^{\infty} \frac{1}{n^3}$ p-test pass.

$x = -\frac{1}{10} \to \sum \frac{(-1)^n}{n^3}$ converges absolutely

$$\boxed{[-\frac{1}{10}, \frac{1}{10}] = IC}$$

(13) $\sum_{n=2}^{\infty} (-1)^n \frac{x^n}{4^n \ln(n)}$

$$\boxed{\text{So } IC = (-4, 4]}$$

$$\left| \frac{x^{n+1}}{4^{n+1} \ln(n+1)} \cdot \frac{4^n \ln(n)}{x^n} \right| = \left| \frac{\ln(n)}{\ln(n+1)} \right| \left| \frac{x}{4} \right| \xrightarrow{n \to \infty} \frac{|x|}{4} \ \text{want} < 1$$

$$\Rightarrow \boxed{R = 4}$$

$x = -4$: $\sum_{2}^{\infty} (-1)^n \frac{(-4)^n}{4^n \ln(n)} = \sum_{2}^{\infty} \frac{1}{\ln(n)} \not\to$

202 §12.8 #s 16, 19, 22, 25, 28, 32, 33

16 $\sum_{n=0}^{\infty} (-1)^n \frac{(x-3)^n}{2n+1}$

$\left| \frac{(x-3)^{n+1}}{2(n+1)+1} \cdot \frac{2n+1}{(x-3)^n} \right| = \left| \frac{2n+1}{2n+3} \right| |x-3| \xrightarrow{n\to\infty} |x-3| \overset{\text{Want}}{<} 1$

$\boxed{R=1}$

$x=4$: $\sum_{n=0}^{\infty} (-1)^n \frac{(1)^n}{2n+1}$ converges (Alternating)

$x=2$: $\sum_{n=0}^{\infty} \frac{(-1)^n (-1)^n}{2n+1}$ Diverges

$\boxed{IC = (2, 4]}$

19 $\sum_{n=0}^{\infty} \frac{(x-2)^n}{n^n}$ $\boxed{R=\infty,\ IC = (-\infty, \infty)}$

(Thinking n^n is bigger than $n!$ & $\sum \frac{x^n}{n!}$ is very nice)

$\sqrt[n]{\frac{(x-2)^n}{n^n}} = \left| \frac{x-2}{n} \right| \xrightarrow{n\to\infty} 0$ ✓

202 §12.8 #s 22, 25, 28, 32, 33

22 $\sum_{n=1}^{\infty} \frac{n(x-4)^n}{n^3+1}$

(16) $R=1$, $IC=[2,4]$

$\left| \frac{(n+1)(x-4)^{n+1}}{(n+1)^3+1} \cdot \frac{n^3+1}{n(x-4)^n} \right| \xrightarrow{n\to\infty} |x-4| \underset{<}{\text{want}} 1$

$\Rightarrow \boxed{R=1}$

$x=5$: $\sum_{n=1}^{\infty} \frac{n(1)^n}{n^3+1}$ converges (compare to $\sum \frac{1}{n^2}$)

$x=3$: $\sum_{n=1}^{\infty} (-1)^n \cdot \frac{n}{n^3+1}$ converges.

$\boxed{IC=[3,5]}$

25 $\sum_{n=1}^{\infty} \frac{(4x+1)^n}{n^2}$ $\left| \frac{(4x+1)^{n+1}}{(n+1)^2} \cdot \frac{n^2}{(4x+1)^n} \right| \xrightarrow{n\to\infty} |4x+1| < 1$

$\overset{\text{want}}{\Longrightarrow} 4|x+\frac{1}{4}| < 1 \overset{\text{want}}{\Longrightarrow} |x+\frac{1}{4}| < \boxed{\frac{1}{4} = R}$

$x=0$: $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges (p-test) $|x-(-\frac{1}{4})|$

$x=-\frac{1}{2}$: $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ converges. So, $IC=[-\frac{1}{2}, 0]$

202 §12.8 #s 28, 32, 33

28 $\displaystyle\sum_{n=1}^{\infty} \frac{n!\,x^n}{1\cdot 3\cdot 5\cdots(2n-1)}$

$$\left| \frac{(n+1)!\,x^{n+1}}{1\cdot 3\cdot 5\cdots(2n-1)(2(n+1)-1)} \cdot \frac{1\cdot 3\cdot 5\cdots(2n-1)}{n!\,x^n} \right|$$

$$= \left| \frac{(n+1)x}{2n+1} \right| \xrightarrow{n\to\infty} \frac{1}{2}|x| < 1 \Longrightarrow |x| < 2$$

$$\boxed{R=2}$$

$x = 2$: $\displaystyle\sum_{n=1}^{\infty} \frac{n!\,2^n}{1\cdot 3\cdot 5\cdots(2n-1)}$

$$\frac{n!\,2^n}{1\cdot 3\cdot 5\cdots(2n-1)} = \frac{(n(n-1)\cdots(3)(2)(1))(2^n)}{1\cdot 3\cdot 5\cdots(2n-1)}$$

$$= \frac{2\cdot 4\cdot 6\cdots(2n)}{1\cdot 3\cdot 5\cdots(2n-1)} > 2\cdot 1 = 2 \quad \forall n \Longrightarrow$$

$\lim\limits_{n\to\infty} a_n \geq 2 \neq 0$ Diverges.

Same deal for $|a_n|$

202 §12.8 #s 32, 33

(32) Let p & q be real numbers w/ $p<q$. Find a power series whose interval of convergence is

(a) (p,q) midpt: $\frac{p+q}{2}$

$$\sum_{n=0}^{\infty} \frac{\left(x-\frac{p+q}{2}\right)^n}{(q-p)^n}$$

Geometric:

(b) $(p,q]$: $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{x-\frac{p+q}{2}}{q-p}\right)^n}{n+1}$ See #4

(c) $[p,q)$: $\displaystyle\sum_{n=1}^{\infty} \frac{\left(\frac{x-\left(\frac{p+q}{2}\right)}{q-p}\right)^n}{\sqrt{n}}$ SEE #2

(d) $[p,q]$: $\displaystyle\sum_{n=1}^{\infty} \frac{\left(\frac{x-\frac{p+q}{2}}{q-p}\right)^n}{n^3}$ SEE #10

(33) Is it possible to find a power series whose IC is $[0,\infty)$?

No. Power series have IC's that are symmetric about some center. So, $(-\infty,\infty)$ is the only option for an infinite IC.