202 §12.5 #s 6, 7, 11-19, 22, 24, 29, 31-34

#s 2-20 Test for convergence

6 $\sum_{n=1}^{\infty}(-1)^{n-1}\cdot\frac{1}{\ln(n+4)}$

$\ln(x)$ is increasing $\forall\, x>0$
$\ln(x+4)$ " " " $x>-4$

$\lim_{n\to\infty}\frac{1}{\ln(n+4)}=0 \Longrightarrow \boxed{\text{Convergent.}}$ Alternating Series

7 $\sum_{n=1}^{\infty}(-1)^{n}\frac{3n-1}{2n+1}$. Test for divergence: $a_n \xrightarrow{n\to\infty} \frac{3}{2}$

$\boxed{\text{Divergent}}$

12 $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{e^{\frac{1}{n}}}{n}$ $e^{\frac{1}{n}}$ is decreasing
n is increasing

#18 $\therefore \frac{e^{\frac{1}{n}}}{n}$ is decreasing.

$e^{\frac{1}{n}}$ is bdd above by e^{1}.

$n \xrightarrow{n\to\infty} \infty$, duh. $\therefore \frac{e^{\frac{1}{n}}}{n} \xrightarrow{n\to\infty} 0$

$\therefore$ Series converges by Alternating Series Test.

14 $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\ln(n)}{n}$ n grows faster than $\ln(n)$
so $\frac{\ln(n)}{n}$ is decreasing.

$\lim_{n\to\infty}\frac{\ln(n)}{n} \overset{L'H}{=} \lim_{n\to\infty}\frac{\frac{1}{n}}{1}=0.$ Converges

$\frac{d}{dn}[a_n]=\frac{\frac{1}{n}\cdot n-1\ln(n)}{n^2}<0$

when $\ln(n)>1$.
when $n>e$

202 § 12.5 #s 15–19, 22, 24, 29, 31–34

16 $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{\sin\left(\frac{n\pi}{2}\right)}{n!}$

$= (-1)^0 \frac{\sin\left(\frac{\pi}{2}\right)}{1!} + (-1)^1 \frac{\sin \frac{2\pi}{2}}{2!} + (-1)^2 \frac{\sin\left(\frac{3\pi}{2}\right)}{3!} + \cdots$

$= 1 - 0 + \frac{(-1)}{3!} - 0 + (-1)^4 \frac{\sin\left(\frac{5\pi}{2}\right)}{5!} + \cdots$

$= 1 - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!} + \cdots$

$= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{(2n-1)!}$ $\boxed{\text{Converges}}$

① Decreasing
② $\xrightarrow{n\to\infty} 0$
③ Alternating

18 $\sum_{n=1}^{\infty} (-1)^n \cos\left(\frac{\pi}{n}\right) = -\cos\pi + \cos\frac{\pi}{2} - \cos\frac{\pi}{3} + \cos\frac{\pi}{4}$

$-\cos\frac{\pi}{5} + \cos\frac{\pi}{6} - \cos\frac{\pi}{7} + \cdots$

$a_n \xrightarrow{n\to\infty} \cos(0) = 1$ $\boxed{\text{Diverges}}$

22 Calculate $s_1, s_2, \ldots, s_{10}$ and graph. Yuck

24 Show $\sum a_k$ converges. How many terms to obtain desired accuracy?

$\sum_{n=1}^{\infty} (-1)^n \frac{1}{n5^n}$ Passes Alternating Series Test ✓

a_n decreasing, $a_n \xrightarrow{n\to\infty} 0$.

$|\text{error}| < .0001$ Want $a_{n+1} < .0001$ since $a_5 = .000064 < .0001$,

$\boxed{S_4 \text{ is good!!}}$

202 § 12.5 #s 31–34

(31) Is the 50^{th} partial sum an over- or underestimate of $S = \sum_{n=1}^{\infty}(-1)^{n-1}\cdot\frac{1}{n}$?

$$S_{50} = \sum_{n=1}^{50}(-1)^{n-1}\left(\frac{1}{n}\right) = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{49} - \frac{1}{50}$$

is an <u>under</u> estimate, since $a_{50} = -\frac{1}{50}$ is negative. The next term $a_{51} = \frac{1}{51}$ will take it above, the next below, and so forth.

#s 32–34 For what values of p is the series convergent?

(32) $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^p}$. Need $\boxed{p \geq 1}$ so that $a_n \xrightarrow{n\to\infty} 0$.

(34) $\sum_{n=2}^{\infty}(-1)^{n-1}\frac{\ln(n)^p}{n}$ Alternates. Need decreasing and need $a_n \xrightarrow{n\to\infty} 0$.

(i) $p>1$ $f(x) = \frac{\ln(x)^p}{x} \Rightarrow f'(x) = \frac{p\ln(x)^{p-1} \times - \ln(x)}{}$

$$= \ln(x)^{p-1}\left[\frac{px - \ln(x)}{x^2}\right] \overset{\text{SET}}{=} 0 \Rightarrow$$

$x = \frac{\ln(x)}{p}$ and x eventually is greater than $\frac{\ln(x)}{p}$.

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#34 cont'd

$$f(x) = \frac{(\ln(x))^p}{x}$$

(i) $p \leq 0$, then $\frac{\ln(x)^p}{x}$ is of the form $\frac{1}{\ln(x)^{-p} x}$ where $-p \geq 0$. In that case, $\ln(x)^{-p}$ and x are both increasing, so $\frac{1}{\ln(x)^{-p} x}$ is decreasing and $\frac{1}{\ln(x)^{-p} x} \xrightarrow{x \to \infty} 0$, so it converges. $p \leq 0 \Rightarrow$ Convergent

(ii) $0 < p < 1$

$$f'(x) = \frac{p \ln(x)^{p-1} \cdot \frac{1}{x} \cdot x - \ln(x)^p}{x^2}$$

$$= \frac{\ln(x)^{p-1}\left[p - \ln(x)\right]}{x^2} \overset{\text{SET}}{=} 0 \Rightarrow \ln(x) = p$$

$$\Rightarrow x = e^p$$

and for $x > e^p$, we have $p - \ln(x) < 0$.

Since $\ln(x)$ is eventually > 0 and $x^2 > 0$, there, we see that $f(x)$ eventually is decreasing.

Also, $\frac{\ln(x)^p}{x} \xrightarrow{x \to \infty} 0$, since x is eventually going to overwhelm the $\ln(x)^p$. $0 < p < 1$ converges

202 §12.5 #34

#34 cont'd

(iii) $p=1$ $\quad \frac{\ln(x)}{x} \xrightarrow{x\to\infty} 0$ ✓

$$f'(x) = \frac{\frac{1}{x}\cdot x - \ln(x)}{x^2} = \frac{1-\ln(x)}{x^2}$$ eventually

is negative (for $x > e$) ✓

So it converges for $p=1$

$p > 1$: $f'(x) = \dfrac{p\ln(x)^{p-1}\cdot\frac{1}{x}\cdot x - \ln(x)^p}{x^2}$

$= \ln(x)^{p-1}\left[\dfrac{p-\ln(x)}{x^2}\right]$ is eventually negative,

so eventually, $f(x)$ is decreasing

Finally $\lim\limits_{x\to\infty} \dfrac{\ln(x)^p}{x} \overset{L'H}{=} \lim\limits_{x\to\infty} \dfrac{p\ln(x)^{p-1}\cdot\frac{1}{x}}{1}$

$= \lim\limits_{x\to\infty} \dfrac{p\ln(x)^{p-1}}{x} \overset{L'H}{=} \dfrac{p(p-1)\ln(x)^{p-2}}{x} \overset{L'H}{=} \cdots$

$\overset{L'H}{=} \dfrac{p(p-1)\cdots\ln(x)^{p-k}}{x}$ Eventually the positive powers are exhausted and we have $\dfrac{p(p-1)\cdots}{\ln(x)^{\text{Positive}}\, x}$ which converges to zero. So $p \geq 1$ converges

$$\boxed{\text{CONVERGES } \forall\, p \in \mathbb{R}}$$