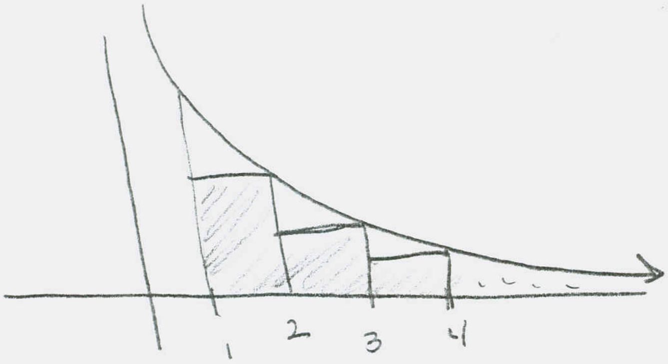
202 §12.3 #s 1, 2, 3, 7, 8, 9-12, 15, 20, 30, 32, 35

#s 2, 10, 20, 32

(1)



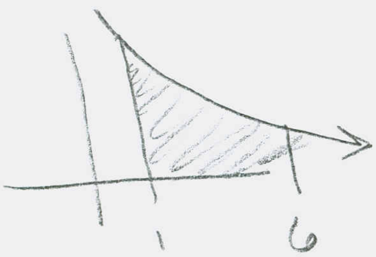
$\sum_{n=2}^{\infty} \frac{1}{n^{1.3}}$ = sum of shaded rectangles

$\int_{1}^{\infty} \frac{1}{x^{1.3}}\,dx$ = Area under $f(x) = \frac{1}{x^{1.3}}$

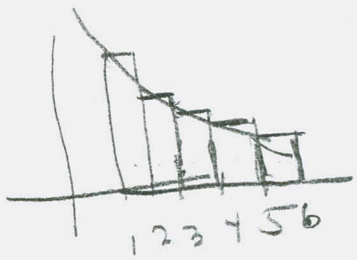
(2) Suppose f is cont^{s}, $f > 0$, f decreasing $\forall\ x \geq 1$ & $a_n = f(n)$. Draw pics to rank the following:

#10

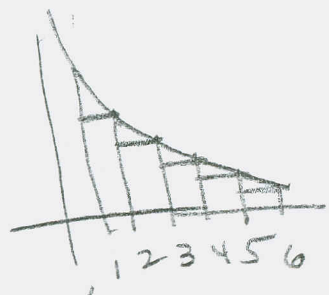
$\int_{1}^{6} f(x)\,dx \qquad \sum_{k=1}^{5} a_k \qquad \sum_{k=2}^{6} a_k$



$\int_{1}^{6} f(x)\,dx$



$\sum_{k=1}^{5} a_k$



$\sum_{k=2}^{6} a_k$

$$\boxed{\sum_{k=2}^{6} a_k < \int_{1}^{6} f(x)\,dx < \sum_{k=1}^{5} a_k}$$

202 §12.3 #s 3, 7–12, 15, 20, 30, 32, 35

#s 3–8 Use Integral Test to determine convergence props of the given series.

(8) $\sum_{n=1}^{\infty} \frac{n+2}{n+1} = \sum_{n=1}^{\infty}\left(1+\frac{1}{n+1}\right)$ Integral test applies ✓

$$\Rightarrow \int_1^{\infty}\left(1+\frac{1}{x+1}\right)dx = \lim_{t\to\infty}\left[x-\frac{1}{(x+1)^2}\right]_1^t = \infty \Rightarrow \boxed{\text{Divergent}}$$

#s 9–26 Determine convergence/divergence

(9) $\sum_{n=1}^{\infty} \frac{2}{n^{.85}}$ $0<p<1 \Rightarrow \boxed{\text{Divergent}}$

(10) $\sum_{n=1}^{\infty}\left(n^{-1.4}+3n^{-1.2}\right)$ $\boxed{\text{Converges,}}$ since the two series $\sum n^{-1.4}$ and $\sum 3n^{-1.2}$ both pass p-test.

(12) $1+\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}+\dots$ $\boxed{\text{Converges.}}$

Re-write : $\sum_{n=1}^{\infty}\frac{1}{n^{3/2}}$ p-test, $p=\frac{3}{2}$

(20) $\sum_{n=1}^{\infty}\frac{1}{n^2-4n+5} = \sum_{n=1}^{\infty}\frac{1}{(n-2)^2+1}$ $f(x)=\frac{1}{(x-2)^2+1}$ is positive, decreasing on $[2,\infty)$

#32

$$\int_2^{\infty}\frac{dx}{(x-2)^2+1} = \lim_{t\to\infty}\left[\arctan(x-2)\right]_2^t = \boxed{\frac{\pi}{2}}$$ (converges)

202 §12.3 #s 30, 32, 35

30 Find $p \ni \sum a_k$ converges

$$\sum_{n=1}^{\infty} \frac{\ln(n)}{n^p}$$

$p \le 0$: Diverges. $\therefore$ Assume $p > 0$

Since $\ln(n) > 1$ for $n > e$, comparison to $\sum_{n=1}^{\infty} \frac{1}{n^p}$ says that $0 < p \le 1$ diverges.

So now, suppose $p > 1$. Then

$$f(x) = \frac{\ln(x)}{x^p} \Longrightarrow f'(x) = \frac{\frac{1}{x} \cdot x^p - \ln(x) p x^{p-1}}{x^{2p}}$$

$$= \frac{x^{p-1}(1 - p\ln(x))}{x^{2p}} \overset{\text{SET}}{=} 0 \Longrightarrow p\ln(x) = 1$$

$$\Longrightarrow \ln(x) = \frac{1}{p}$$

$$\Longrightarrow x = e^{\frac{1}{p}} \text{ and}$$

for $x > e^{1/p}$, $f'(x) < 0$. Integral Test applies.

$$\int_1^{\infty} \frac{\ln(x)}{x^p}\,dx = \lim_{t \to \infty} \left[\frac{x^{1-p}[(1-p)\ln(x) - 1]}{(1-p)^2} \right]_1^t$$

$$= \frac{1}{(1-p)^2} \left[\lim_{t \to \infty} \left(t^{1-p}(1-p)\ln(t) - 1 \right) - 1(-1) \right]$$

$\Longrightarrow \boxed{\text{Converges.}}$ L'Hôpital may help with $\lim_{t \to \infty} \frac{\ln(t)}{t^{p-1}}$.

202 §12.3 Scratch for #30

$u = \ln(x) \qquad du = \frac{1}{x}\,dx$

$dv = x^{-p} \qquad v = \frac{x^{-p+1}}{-p+1}$

$$uv - \int v\,du = \frac{x^{1-p}}{1-p}\ln(x) - \int \frac{x^{1-p}}{1-p}\cdot\frac{1}{x}\,dx$$

$$= \frac{x^{1-p}}{1-p}\ln(x) - \frac{1}{1-p}\int x^{-p}\,dx$$

$$= \frac{x^{1-p}}{1-p}\ln(x) - \frac{1}{1-p}\cdot\frac{x^{1-p}}{1-p} + C$$

$$= \frac{(1-p)x^{1-p}\ln(x) - x^{1-p}}{(1-p)^2} = \frac{x^{1-p}\left[(1-p)\ln(x) - 1\right]}{(1-p)^2}$$

202 $\S$ 12.3 #s 32, 35

32 (a) Use s_{10} to approximate $\sum_{n=1}^{\infty} \frac{1}{n^4}$.

Estimate the error.

$$S_{10} = \frac{1}{1^4} + \frac{1}{2^4} + \cdots + \frac{1}{10^4} \approx 1.082037$$

$$R_{10} \le \int_{10}^{\infty} \frac{dx}{x^4} = \int_{10}^{\infty} x^{-4}\,dx = \lim_{t\to\infty} \left[-\frac{1}{3}x^{-3}\right]_{10}^{t}$$

$$= -\frac{1}{3}(10^{-3}) = \frac{1}{3000} = \boxed{.000\overline{3}} \approx \text{Error (ceiling)}$$

(b) Use eq'n (3) to get better estimate for the sum.

$$s_{10} + \int_{11}^{\infty} \frac{dx}{x^4} \le S \le s_{10} + \int_{10}^{\infty} \frac{dx}{x^4} = s_{10} + \frac{1}{3000}$$

$$\left(\int_{11}^{\infty} \frac{dx}{x^4} = \frac{1}{3}(11^{-3}) \right)$$

So,

$$\frac{(1.082037 + .000250438267) + (1.082037 + .000\overline{3})}{2}$$

$$\approx 1.082328886 \approx \boxed{1.08233}$$

Error < .00005

(c) $n > 32$

202 §12.3 #s 32, 35

(c) Find $n \ni s_n$ is within .00001 of $\sum$

$R_n \leq \int_n^\infty \frac{dx}{x^4} = \frac{1}{3n^3} \overset{\text{want}}{<} .00001 \Rightarrow$

$3n^3 > 10^5 \Rightarrow n^3 > \frac{10^5}{3} \Rightarrow n > \sqrt[3]{\frac{10^5}{3}}$

≈ 32.18298, so $\boxed{n > 32 \text{ does it!}}$