202 §12.2 #5 1-3, 7-12, 19, 20, 22, 25, 29-31, 35, 36, 41, 42, 47, 48, 52, 65, 69, 70

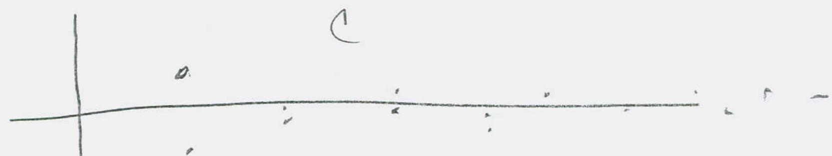
① (a) A sequence is an infinite "list" of real numbers; whereas a series is the <u>sum</u> of a sequence.

② To say that $\sum_{n=1}^{\infty} a_n = 5$ is to say that the sequence $\{s_n\} = \left\{\sum_{k=1}^{n} a_k\right\}$ converges to 5, which is to say, given any $\varepsilon > 0$, $\exists N \in \mathbb{N} \ni$

$|s_n - 5| < \varepsilon \quad \forall n > N.$

① (b) A convergent series is a series in which the sequence of partial sums $\{s_n\}$ converges to a real number. A divergent series, not so much.

③ $\sum_{n=1}^{\infty} \frac{12}{(-5)^n}$



S_ns are all negative, converging rapidly to

$a_1 = -\frac{12}{5}$

$a_2 = \frac{12}{25}$

$a_3 = \frac{12}{-125}$

$a_4 = \frac{12}{625}$

$a_5 = -\frac{12}{3125}$

$$\frac{12}{-5} \cdot \left(\frac{1}{1+\frac{1}{5}}\right) = -\frac{12}{5}\left(\frac{1}{\frac{6}{5}}\right) = -\frac{12}{5}\left(\frac{5}{6}\right)$$

$$\boxed{= -2}$$

202 §12.2 #s 7-12, 19, 20, 22, 25, 29-31, 35, 36, 41, 42,
47, 48, 52, 65, 69, 70

#s 3-7 Bleah

9 $a_n = \frac{2n}{3n+1}$

(a) $\{a_n\}$ converges to $\frac{2}{3}$

(b) $\sum a_n$ does not converge, since $\frac{2}{3} \neq 0$

10 (a) No difference between
$\sum_{i=1}^{n} a_i$ and $\sum_{j=1}^{n} a_j = a_1 + a_2 + \cdots + a_n$

(b) $\sum_{i=1}^{n} a_i = a_1 + a_2 + \cdots + a_n$, but $\sum_{i=1}^{n} a_j = \underbrace{a_j + a_j + \cdots + a_j}_{n \text{ of 'em.}}$

#s 11-20 Convergent or divergent? If convergent, find sum.

11 $3 + 2 + \frac{4}{3} + \frac{8}{9} + \cdots = 3\left(\frac{2}{3}\right)^0 + 3\left(\frac{2}{3}\right) + 3\left(\frac{2}{3}\right)^2 + \cdots$

$$\sum_{n=1}^{\infty} 3\left(\frac{2}{3}\right)^{n-1} = \frac{3}{1-\frac{2}{3}} = \frac{3}{\frac{1}{3}} = \boxed{9}$$

12 $\frac{1}{8} - \frac{1}{4} + \frac{1}{2} - 1 = \frac{1}{8}(2)^0 - \frac{1}{8}(2)^1 + \frac{1}{8}(2)^2 - \frac{1}{8}(2)^3 + \cdots$

$= \sum_{n=1}^{\infty} \frac{1}{8}(-2)^{n-1}$ Diverges $(|r| > 1)$

19 $\sum_{n=0}^{\infty} \frac{\pi^n}{3^{n+1}} = \sum_{n=1}^{\infty} \frac{\pi^{n-1}}{3^n} = \sum_{n=1}^{\infty} \frac{1}{3} \cdot \left(\frac{\pi}{3}\right)^{n-1}$ Diverges. $r > 1$

20 $\sum_{n=1}^{\infty} \frac{e^n}{3^{n-1}} = \sum_{n=1}^{\infty} e \cdot \frac{e^{n-1}}{3^{n-1}} = \frac{e}{1-\frac{e}{3}} = \frac{e}{\frac{3-e}{3}} = \boxed{\frac{3e}{3-e}}$

#42

202 S'12, 2 #s 22, 25, 29–31, 35, 36, 41, 42, 47, 48, 52, 65, 69, 70

#s 21–34 Converge or Diverge? If convergent, find the sum.

(22) $\sum_{n=1}^{\infty} \frac{n+1}{2n-3}$ Diverges. $\left(a_n \xrightarrow{n\to\infty} \frac{2}{3} \neq 0\right)$

(25) $\sum_{n=1}^{\infty} \frac{1+2^n}{3^n} = \sum_{n=1}^{\infty}\left(\left(\frac{1}{3}\right)^n + \left(\frac{2}{3}\right)^n\right) = \sum_{n=1}^{\infty}\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)^{n-1} + \sum_{n=1}^{\infty}\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)^{n-1}$

$= \frac{\frac{1}{3}}{1-\frac{1}{3}} + \frac{\frac{2}{3}}{1-\frac{2}{3}} = \frac{\frac{1}{3}}{\frac{2}{3}} + \frac{\frac{2}{3}}{\frac{1}{3}} = \frac{1}{3}\cdot\frac{3}{2} + \frac{2}{3}\cdot\frac{3}{1} = \boxed{\frac{5}{2}}$

(29) $\sum_{n=1}^{\infty} \ln\left(\frac{n^2+1}{2n^2+1}\right)$ Diverges $\left(a_n \xrightarrow{n\to\infty} \ln\left(\frac{1}{2}\right) \neq 0\right)$

(30) $\sum_{n=1}^{\infty} \cos(1)^n = \sum_{n=1}^{\infty} \cos(1)\left(\cos(1)\right)^{n-1} = \boxed{\frac{\cos(1)}{1-\cos(1)}}$

(31) $\sum_{n=1}^{\infty} \arctan(n)$ Diverges $\left(a_n \xrightarrow{n\to\infty} \frac{\pi}{2} \neq 0\right)$

#s 35–40 use telescoping. As before, if convergent, find the sum.

(35) $\sum_{n=2}^{\infty} \frac{2}{n^2-1}$

$2 = A(n+1) + B(n-1)$

$2 = A - B \quad = -B - B = -2B \implies B = -1$

$0 = A + B \implies A = 1$

$= \sum_{n=2}^{\infty}\left(\frac{1}{n-1} - \frac{1}{n+1}\right) = 1 - \frac{1}{3} + \frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \frac{1}{5} + \frac{1}{4} - \frac{1}{6} + \cdots$

$\sum_{n=2}^{4} a_n = 1 - \frac{1}{3} + \frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \frac{1}{5} = \frac{1}{2} + 1 + \frac{1}{n-2} - \frac{1}{n} + \frac{1}{n-1} - \frac{1}{n+1}$

$\sum_{k=2}^{n} a_k \xrightarrow{n\to\infty} \boxed{\frac{3}{2}}$

202 §12.2 #s 36, 41, 42, 47, 48, 52, 65, 69, 70

(36) $\sum_{n=1}^{\infty} \frac{2}{(n^2+4n+3)}$ $\quad (n+3)(n+1)$

$2 = A(n+1) + B(n+3)$

$2 = A + 3B = A - 3A = -2A \Rightarrow A = -1$

$0 = A + B \quad \Rightarrow B = +1$

$= \sum_{n=1}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n+3} \right)$

$\sum_{k=1}^{6} a_k = \frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \frac{1}{5} + \frac{1}{4} - \frac{1}{6} + \frac{1}{5} - \frac{1}{7} + \frac{1}{6} - \frac{1}{9}$

$= \frac{1}{2} + \frac{1}{3} - \frac{1}{n-1} - \frac{1}{n-3} \xrightarrow{n \to \infty} \boxed{\frac{5}{6}}$

#s 41-46 Express as a ratio of integers

OLD

(41) $.2222\ldots = x$

$2.2222\ldots = 10x$

$2 = 9x \Rightarrow \boxed{x = \frac{2}{9}}$

NEW:

$.2 + (.1)(.2) + (.1)^2(.2) + \ldots$

$= \frac{.2}{1-.1} = \frac{.2}{.9} = \frac{2}{9}$

(42) $.\overline{73} = .7373\ldots = x$

$73.7373\ldots = 100x$

$73 = 99x \Rightarrow \boxed{x = \frac{73}{99}}$

#48

$.73 + (.01)(.73) + (.01)^2(.73) + \ldots$

$= \frac{.73}{1-.01} = \frac{.73}{.99} = \boxed{\frac{73}{99}}$

#s 47-51 Find $x \ni$ series converges. Find sum (in terms of x?) Need $\boxed{|x| < 3}$ and then

(47) $\sum_{n=1}^{\infty} \frac{x^n}{3^n} = \sum_{n=1}^{\infty} \frac{x}{3} \cdot \left(\frac{x}{3}\right)^{n-1}$

$S_n \xrightarrow{n \to \infty} \frac{\frac{x}{3}}{1 - \frac{x}{3}} = \frac{x}{3} \cdot \frac{3}{3-x} = \boxed{\frac{x}{3-x}}$

202 §12.2 #s 48, 52, 65, 69, 70

48 $\sum_{n=1}^{\infty} (x-4)^n = \sum_{n=1}^{\infty} (x-4)\cdot(x-4)^{n-1}$ Need $|x-4|<1$

$-1 < x-4 < 1$

$3 < x < 5 \Rightarrow x \in (3,5)$

$= \frac{x-4}{1-(x-4)} = \frac{x-4}{1-x+4} = \boxed{\frac{x-4}{5-x}}$

52 IN CLASS

65 What's wrong with

$0 = 0+0+0+\cdots$

$= (1-1)+(1-1)+(1-1)+\cdots$

$= 1-1+1-1+1-1$

$= 1+(-1+1)+(-1+1)+(-1+1)+\cdots$

$= 1+0+0+\cdots = 1$

By our definition, $1-1+1-1+1-1\cdots$ is geometric series with $r=-1$. The partial sums oscillate between 0 & 1, depending if n is odd or even.

There is more in higher math on when it is or is not appropriate to re-group a series. Some SURPRISING results about re-grouping alternating series, in particular.

202 §12.2 #s 69, 70

(69) $\sum b_n$ divergent & $\sum a_n$ convergent $\Longrightarrow$

$\sum(a_n+b_n)$ is divergent.

$\boxed{\text{Pf}}$ (In class)

suppose $\sum a_n$ & $\sum(a_n+b_n)$ are convergent,

Then $\sum((a_n+b_n)-a_n) = \sum b_n$ is convergent $\times$ ∎

(70) If $\sum a_n$ & $\sum b_n$ diverge, is $\sum(a_n+b_n)$ divergent?

No. $\sum a_n = \sum 1$, $\sum b_n = \sum -1$ $\longrightarrow$

$\sum(a_n+b_n) = \sum 0 = 0$

202 §12.2 #52

52 We show that $\sum \ln(1+\frac{1}{n})$ is a divergent series.

$\ln(1+\frac{1}{n}) = \ln(\frac{n+1}{n}) = \ln(n+1) - \ln(n)$, so

$\sum_{k=1}^{n} \ln(1+\frac{1}{k}) = \ln(2) - \ln(1) + \ln(3) - \ln(2) +$

$\ldots + \ln(n+1) - \ln(n) = \ln(n+1).$

So $\{S_n\}$ does not converge, since it's increasing without bound, just as $\ln(n)$ increases without bound.