202 §12.1 #s 1,2,3-53 ODDS, 56, 57, 63, 67
56? 67!

① (a) A sequence is a function from $\mathbb{N}$ into $\mathbb{R}$

(b) $\lim_{n\to\infty} a_n = 8$ means, given any $\varepsilon > 0$, $\exists N \in \mathbb{N}$

$\ni \quad |a_n - 8| < \varepsilon \;\; \forall \; n > N$.

(c) $\lim_{n\to\infty} a_n = \infty$ means, given any $M > 0$, $\exists N \in \mathbb{N}$

$\ni \quad a_n > M \;\; \forall \; n > N$.

② (a) A convergent sequence is a sequence $\ni$ $\lim_{n\to\infty} a_n$ exists, $a_n = \frac{1}{n}$, $a_n = \frac{n}{n+1}$

(b) A divergent sequence is a sequence $\ni$ $\lim_{n\to\infty} a_n \not\exists$.
$a_n = n$, $a_n = \sin(n)$

#s 3-8 1st 5 terms

③ $a_n = 1 - .2^n$

$a_1 = .8$, $a_2 = .96$, $a_3 = .992$, $a_4 = .9984$, $a_5 = .99968$

⑤ $a_n = \dfrac{3(-1)^n}{n!}$ $\quad -3, 1.5, -.5, .125, -.025$

⑦ $a_n = 3$, $a_{n+1} = 2a_n - 1$ $\quad 3, 5, 9, 17, 33, 65$

~~9~~ Find a formula for #s 9-14

⑨ $\{1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \ldots\}$ $\quad a_1 = \frac{1}{2(1)-1}$, $a_2 = \frac{1}{2(2)-1}$, ... $\boxed{a_n = \frac{1}{2n-1}}$

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11 $\{2, 7, 12, 17, \dots\}$ $\boxed{a_n = 5(n-1)+2 = 5n-3}$

3 $\{1, -\frac{2}{3}, \frac{4}{9}, -\frac{8}{27}, \dots\}$ $a_n = (-1)^{n-1}\left(\frac{2}{3}\right)^{n-1}$

15 First 6 terms of

$a_n = \frac{n}{2n+1}$: $\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{5}{9}, \frac{7}{11}, \frac{9}{13}$

$\lim_{n\to\infty} a_n = \frac{1}{2}$

#s 17-46 Converge/Diverge? Limit?

17 $a_n = 1 - .2^n$ $\boxed{\text{Converges to } 1}$

19 $a_n = \frac{3+5n^2}{n+n^2}$ $\boxed{\text{converges to } 5}$

21 $a_n = e^{\frac{1}{n}}$ $\boxed{\text{converges to } 1}$

$\ln y = \ln(e^{\frac{1}{n}}) = \frac{1}{n} \xrightarrow{n\to\infty} 0 \longrightarrow$

$y = e^0 = \boxed{1 = y = L}$

23 $a_n = \tan\left(\frac{2n\pi}{1+8n}\right) \xrightarrow{n\to\infty} \boxed{\tan\left(\frac{\pi}{4}\right) = 1}$

25 $a_n = \frac{(-1)^{n-1} n}{n^2+1} \xrightarrow{n\to\infty} \boxed{0 = L}$

27 $a_n = \cos\left(\frac{n}{2}\right)$ doesn't converge.

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29 $\dfrac{(2n-1)!}{(2n+1)!} = \dfrac{1}{(2n+1)(2n)} \xrightarrow{n\to\infty} \boxed{0 = L}$

31 $\dfrac{e^n+e^{-n}}{e^{2n}-1} = \dfrac{e^n\left[1+e^{-2n}\right]}{e^n\left[e^n-\frac{1}{e^n}\right]} = \dfrac{1+e^{-2n}}{e^n-\frac{1}{e^n}} \xrightarrow{n\to\infty} \boxed{0=L}$

33 $n^2e^{-n} = a_n = \dfrac{n^2}{e^n} \xrightarrow{L'H} \dfrac{2n}{e^n} \xrightarrow{L'H} \dfrac{2}{e^n} \xrightarrow{n\to\infty} 0$

$= \boxed{L=0}$

35 $a_n = \dfrac{\cos^2(n)}{2^n} \xrightarrow{n\to\infty} \boxed{0=L}$

37 $a_n = n\sin\left(\frac{1}{n}\right) = \dfrac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}} \xrightarrow{n\to\infty} \boxed{1=L}$

39 $a_n = \left(1+\frac{2}{n}\right)^n = \left(1+\dfrac{1}{\left(\frac{n}{2}\right)}\right)^n = \left(1+\dfrac{1}{\left(\frac{n}{2}\right)}\right)^{\frac{n}{2}\cdot 2}$

$\xrightarrow{n\to\infty} \boxed{e^2 = L}$

41 $\ln(2n^2+1) - \ln(n^2+1) = \ln\left(\dfrac{2n^2+1}{n^2+1}\right) \xrightarrow{n\to\infty} L = \boxed{\ln(2)}$

43 $\{0,1,0,0,1,0,0,0,1,0,0,0,0,1,\ldots\}$ Diverges

45 $a_n = \dfrac{n!}{2^n}$ Diverges $\left(> \frac{n}{4} \text{ for } n>1\right)$

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47 #s 47–53 basically suck

56 Find 1^{st} 40 terms

$$a_{n+1} = \begin{cases} \frac{1}{2} a_n & \text{if } a_n \text{ is even} \\ 3a_n + 1 & \text{if } a_n \text{ is odd} \end{cases}$$

$a_1 = 11$: 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1, 4, ... Repeats

$a_1 = 25$, 76, 38, 19, 58, 29, 88, 44, 22, 11, then repeat the above.

Conjecture: Regardless of a_1, eventually the sequence drops into the 4, 2, 1 pattern

57 For what values of r is $n r^n$ convergent?
All values of $r \ni |r| < 1$

63 $a_n = n(-1)^n$ is not monotonic

67 Find the limit: $\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \ldots$

$= 2^{\frac{1}{2}}, \left(2 \cdot 2^{\frac{1}{2}}\right)^{\frac{1}{2}} = \left(2^{\frac{3}{2}}\right)^{\frac{1}{2}} = 2^{\frac{3}{4}}, \sqrt{2 \cdot 2^{\frac{3}{4}}} = \left(2^{\frac{7}{4}}\right)^{\frac{1}{2}} = 2^{\frac{7}{8}}$

$2^{\frac{15}{16}}, \ldots, 2^{\frac{2^n - 1}{2^n}} \xrightarrow{n \to \infty} 2$