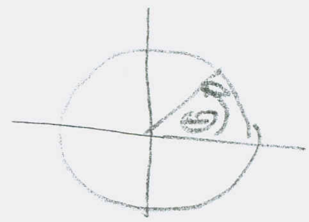
202 §11.4 #s 1, 4, 7, 10, 13, 15, 20, 26, 31, 35, 39,
42, 45, 52.

Area of circle is πr^2

Area is proportional to the angle



$A = k\theta$

$A = \pi r^2$ when $\theta = 2\pi$

$\pi r^2 = k\theta = k \cdot 2\pi$, so

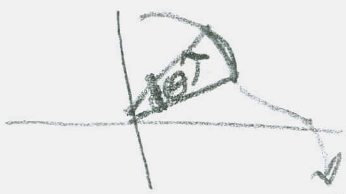
$k = \frac{r^2}{2}$ and so Area of sector is

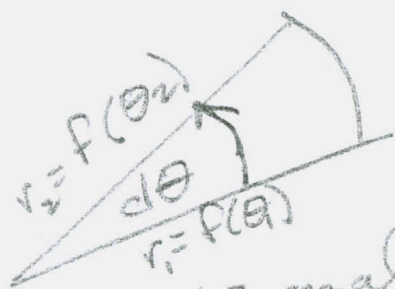
$A = \frac{r^2}{2} \cdot \theta = \frac{1}{2} r^2 \theta$

By proportions: $\frac{\text{Area of circle}}{2\pi \text{ radians}} = \frac{\text{Area of sector}}{\theta \text{ radians}}$.

$\frac{\pi r^2}{2\pi} = \frac{\text{Area of sector}}{\theta}$

Area of sector $= \frac{1}{2} r^2 \theta$





when $d\theta$ small, this is close to area of $d\theta$ wedge.

#s 1-4 Find area of region bdd by curve, inside the given sector.

4 $r = \sqrt{\sin\theta}$, $0 \le \theta \le \pi$

$\frac{1}{2}\int_0^{\pi} \sqrt{\sin\theta}^{\,2}\, d\theta = \frac{1}{2}\int_0^{\pi} \sin\theta\, d\theta$

#10

$= \frac{-\cos\theta}{2}\Big]_0^{\pi} = -\frac{\cos\pi - (-\cos 0)}{2}$

$= -\frac{(-1)+1}{2} = \boxed{\frac{2}{2}}$

$= \boxed{1}$

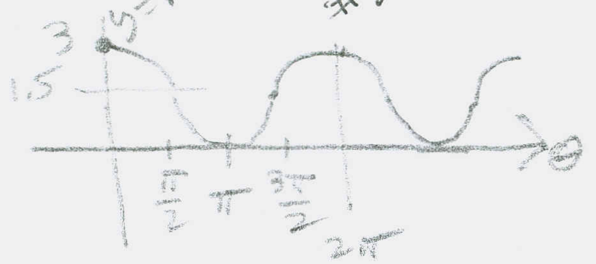
202 §11.4 #5 7, 10, 13, 15, 20, 26, 31, 35, 39, 42, 45, 52

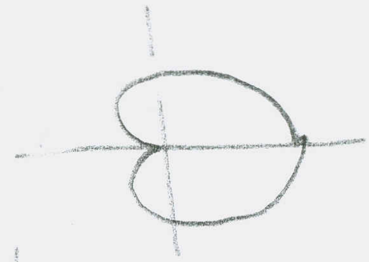
10 Sketch the curve & find the enclosed area.

$r = 3(1+\cos\theta)$

Symmetry: θ by $-\theta$ about polar axis.

#20





Twice the area of top $\frac{1}{2}$.

$$2\cdot\frac{1}{2}\int_0^{\pi}\left(3(1+\cos\theta)\right)^2 d\theta = 9\int_0^{\pi}\left(1+2\cos\theta+\cos^2\theta\right)d\theta$$

$$= 9\int_0^{\pi}\left(\frac{2}{2}+\frac{4\cos\theta}{2}+\frac{1+\cos(2\theta)}{2}\right)d\theta$$

$$= 9\int_0^{\pi}\frac{3+4\cos\theta+\cos(2\theta)}{2}\,d\theta$$

$$= \frac{9}{2}\left[3\theta+4\sin\theta+\frac{1}{2}\sin(2\theta)\right]_0^{\pi}$$

$$= \frac{9}{2}\left[3\pi+4\sin(\pi)+\frac{1}{2}\sin(2\pi)-\left(0+4\sin(0)+\frac{1}{2}\sin(2\cdot 0)\right)\right]$$

$$= \frac{9}{2}\left[3\pi\right] = \boxed{\frac{27}{2}\pi}$$

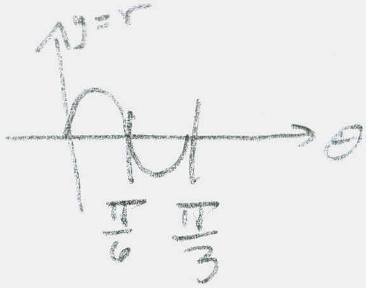
202 §11.4 #s 13, 15, 20, 26, 31, 35, 39, 43, 45, 52

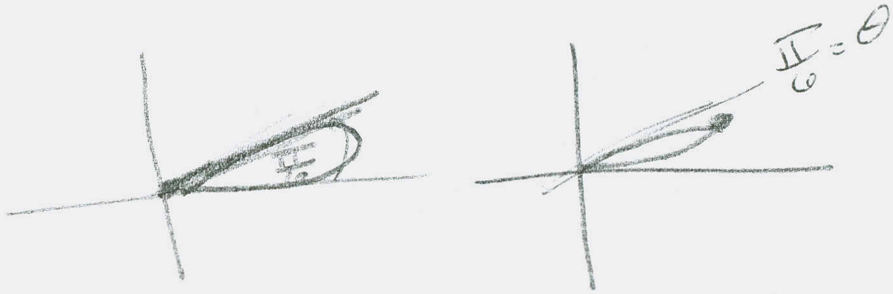
20 Find area enclosed by one loop of the curve.

$r = 2\sin(6\theta)$ $\qquad \frac{2\pi}{6} = \frac{\pi}{3}$ is one ~~loop~~ period

So one loop in $[0, \frac{\pi}{6})$

#26





$$\text{Area} = \frac{1}{2}\int_0^{\frac{\pi}{6}} (2\sin(6\theta))^2 \, d\theta$$

$$= \frac{1}{2}\cdot 4\int_0^{\frac{\pi}{6}} \frac{1-\cos(12\theta)}{2}\, d\theta = \frac{1}{2}\cdot 4 \cdot \frac{1}{2}\int_0^{\frac{\pi}{6}} \left(1 - \cos(12\theta)\right) d\theta$$

$$= \left[\theta - \frac{1}{12}\sin(12\theta)\right]_0^{\frac{\pi}{6}} =$$

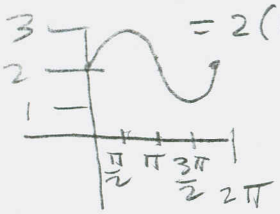
$$= \left[\left(\frac{\pi}{6} - \frac{1}{12}\sin(2\pi)\right) - \left(0 - \frac{1}{2}\sin(0)\right)\right]$$

$$= \frac{\pi}{6} = \boxed{\frac{\pi}{6} = \text{Area}}$$

202 §11.4 #s 26, 31, 35, 39, 42, 45, 52

(26) Find area inside 1st & outside 2nd

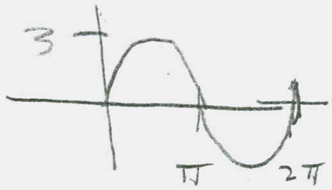
$r = 2 + \sin\theta \quad , \quad r = 3\sin\theta$



$= 2(1 + \frac{1}{2}\sin\theta)$

Cardioid

#42

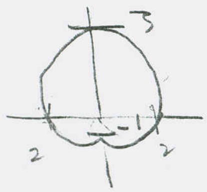


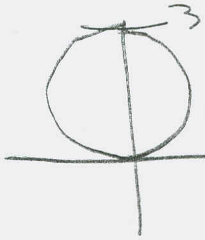
$2 + \sin\theta = 3\sin\theta$

$-2\sin\theta = -2$

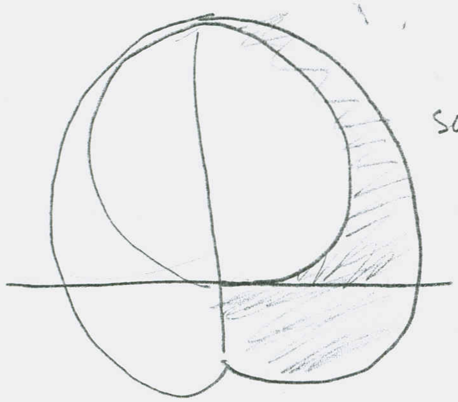
$\sin\theta = 1$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$





is inside the 1st one.



OUTER − INNER

symmetry

$$2 \cdot \frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (2 + \sin\theta)^2 d\theta - 2 \cdot \frac{1}{2}\int_0^{\frac{\pi}{2}} (3\sin\theta)^2 d\theta$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (4 + 2\sin\theta + \sin^2\theta)\, d\theta - \int_0^{\frac{\pi}{2}} 9\sin^2\theta\, d\theta$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 4\, d\theta + 2\overset{\text{ODD}}{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin\theta} + \frac{1}{2}\overset{\text{EVEN}}{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1 - \cos(2\theta))\, d\theta} - \frac{9}{2}\overset{\text{EVEN}}{\int_0^{\frac{\pi}{2}} (1 - \cos(2\theta))\, d\theta}$$

$$= 2 \cdot 4\theta\Big]_0^{\frac{\pi}{2}} + 0 + 2 \cdot \frac{1}{2}\theta\Big]_0^{\frac{\pi}{2}} - 2 \cdot \frac{1}{2} \cdot \frac{1}{2}\sin(2\theta)\Big]_0^{\frac{\pi}{2}} - \frac{9}{2}\Big[\theta - \frac{1}{2}\sin(2\theta)\Big]_0^{\frac{\pi}{2}}$$

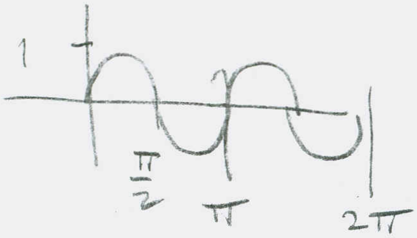
$$= 8 \cdot \frac{\pi}{2} + \frac{\pi}{2} - \frac{1}{2}\underbrace{\Big[\sin(\pi) - \sin(0)\Big]}_{0} - \frac{9}{2}\Big[\frac{\pi}{2} - \frac{1}{2}\sin(\pi) - (0 - \frac{1}{2}\sin(0))\Big]$$

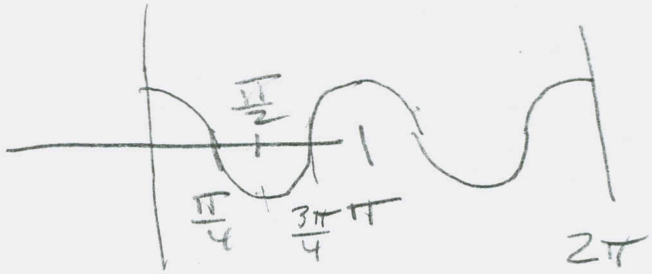
$$= 4\pi + \frac{\pi}{2} - \frac{9\pi}{4} = \frac{16\pi + 2\pi - 9\pi}{4} = \boxed{\frac{9\pi}{4}}$$

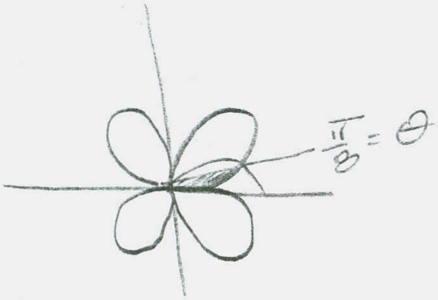
202 §11.4 #5 31, 35, 39, 42, 45, 52

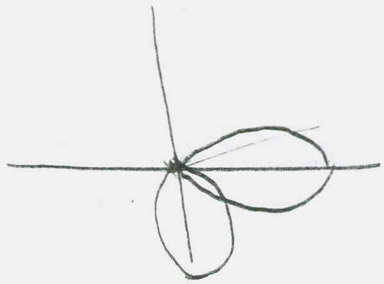
(31) Find area inside both curves.

$r = \sin(2\theta)$ $\qquad\qquad$ $r = \cos(2\theta)$

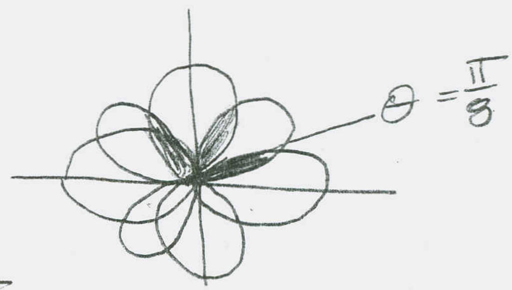








$$\sin(2\theta) = \cos(2\theta)$$
$$\tan(2\theta) = 1$$
$$2\theta = \frac{\pi}{4}, \frac{5\pi}{4}$$
$$\theta = \frac{\pi}{8}, \frac{5\pi}{8}$$



$$\text{Area} = \left[\frac{1}{2}\int_0^{\frac{\pi}{8}} \sin^2(2\theta)\,d\theta + \frac{1}{2}\int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \cos^2(2\theta)\,d\theta\right] \text{TIMES } 8$$

$$= \left[\frac{1}{4}\int_0^{\frac{\pi}{8}} (1-\cos(2\theta))\,d\theta + \frac{1}{4}\int_{\frac{\pi}{8}}^{\frac{\pi}{4}} (1+\cos(2\theta))\,d\theta\right]\cdot 8$$

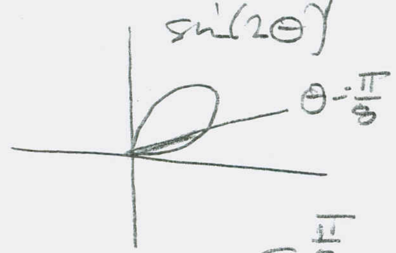
$$= 2\left[\left[\theta - \frac{1}{2}\sin(2\theta)\right]_0^{\frac{\pi}{8}} + \left[\theta + \frac{1}{2}\sin(2\theta)\right]_{\frac{\pi}{8}}^{\frac{\pi}{4}}\right]$$

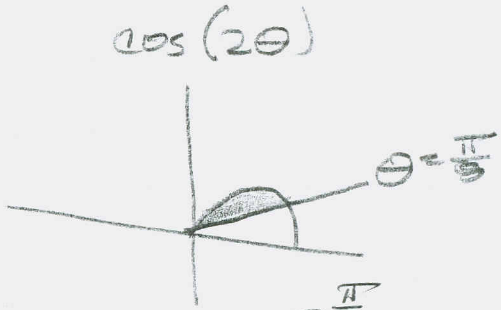
$$= 2\cdot\frac{\pi}{8} - \sin\left(\frac{\pi}{4}\right) + \frac{\pi}{4} + \frac{1}{2}\sin\left(\frac{\pi}{2}\right) - \left(\frac{\pi}{8} + \frac{1}{2}\sin\left(\frac{\pi}{4}\right)\right) =$$

$$\frac{\pi}{4} - \frac{\sqrt{2}}{2} + \frac{\pi}{4} + \frac{1}{2} - \frac{\pi}{8} - \frac{\sqrt{2}}{4} = \frac{3\pi}{8} - \frac{3\sqrt{2}}{4} + \frac{1}{2}$$

202 §11.4 #31 Re-worked.

I think my setup is OK.





Area is $\frac{1}{2}\int_0^{\frac{\pi}{8}} \sin^2(2\theta)\,d\theta$

Area is $\frac{1}{2}\int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \cos^2(2\theta)\,d\theta$

And there are 8 of these little wedges.

$$8\left[\frac{1}{2}\int_0^{\frac{\pi}{8}} \frac{1-\cos(4\theta)}{2}\,d\theta + \frac{1}{2}\int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \frac{1+\cos(4\theta)}{2}\,d\theta\right]$$

$$= 2\left[\left[\theta - \frac{\sin(4\theta)}{4}\right]_0^{\frac{\pi}{8}} + \left[\theta + \frac{\sin(4\theta)}{4}\right]_{\frac{\pi}{8}}^{\frac{\pi}{4}}\right]$$

$$= \left[2\theta - \frac{\sin(4\theta)}{2}\right]_0^{\frac{\pi}{8}} + \left[2\theta + \frac{\sin(4\theta)}{2}\right]_{\frac{\pi}{8}}^{\frac{\pi}{4}}$$

$$= \frac{\pi}{4} - \frac{\sin(\frac{\pi}{2})}{2} - \left(2\cdot 0 - \frac{\sin(0)}{2}\right) + \frac{\pi}{2} + \frac{\sin(\pi)}{2} - \left(\frac{\pi}{4} + \sin(\frac{\pi}{2})\right)$$

$$= \frac{\pi}{4} - \frac{1}{2} + \frac{\pi}{2} + \frac{0}{2} - \frac{\pi}{4} - \frac{1}{2} = \boxed{\frac{\pi}{2} - 1}$$

Book uses symmetry of the wedge and sets it up thus:

$$8 \cdot \frac{1}{2} \cdot 2 \int_0^{\frac{\pi}{8}} \sin^2(2\theta)\,d\theta$$

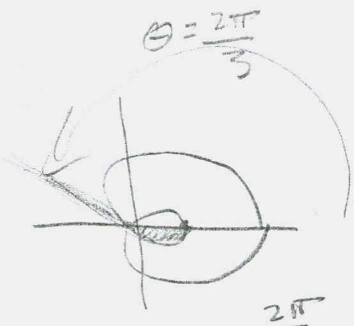
symmetry.

202 §11.4 #s 35, 39, 42, 45, 52

35 Area inside larger loop & outside the smaller for the limaçon

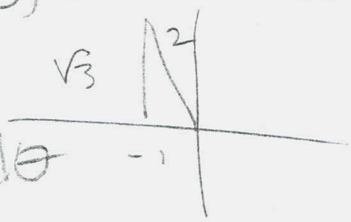
$$r = \frac{1}{2} + \cos\theta = \frac{1}{2}(1 + 2\cos\theta)$$

Bigger than 1 ; Loop



$$1 + 2\cos\theta = 0$$
$$2\cos\theta = -1$$
$$\cos\theta = -\frac{1}{2}$$

Outer: $2 \cdot \frac{1}{2}\int_0^{\frac{2\pi}{3}} \left(\frac{1}{2} + \cos\theta\right)^2 d\theta$



$$\pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

Inner $2 \cdot \frac{1}{2}\int_{\frac{2\pi}{3}}^{\pi} \left(\frac{1}{2} + \cos\theta\right)^2 d\theta$

Area between them: $\int_0^{\frac{2\pi}{3}} \left(\frac{1}{2} + \cos\theta\right)^2 d\theta - \int_{\frac{2\pi}{3}}^{\pi} \left(\frac{1}{2} + \cos\theta\right)^2 d\theta$

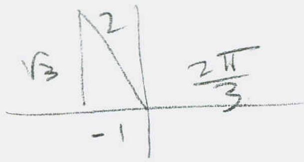
$$= \int_0^{\frac{2\pi}{3}} \left(\frac{1}{4} + \cos\theta + \cos^2\theta\right) d\theta - \int_{\frac{2\pi}{3}}^{\pi} \left(\frac{1}{4} + \cos\theta + \cos^2\theta\right) d\theta = \int_0^{\frac{2\pi}{3}} \left(\frac{1}{4} + \cos\theta + \frac{1 + \cos(2\theta)}{2}\right) d\theta$$

$$- \int_{\frac{2\pi}{3}}^{\pi} \left(\frac{1}{4} + \cos\theta + \frac{1 + \cos(2\theta)}{2}\right) d\theta = \int_0^{\frac{2\pi}{3}} \left(\frac{3}{4} + \cos\theta + \frac{1}{2}\cos(2\theta)\right) d\theta$$

$$- \int_{\frac{2\pi}{3}}^{\pi} \left(\frac{3}{4} + \cos\theta + \frac{1}{2}\cos(2\theta)\right) d\theta$$

202 §11.4 #s 35, 39, 42, 45, 52

35 cont'd



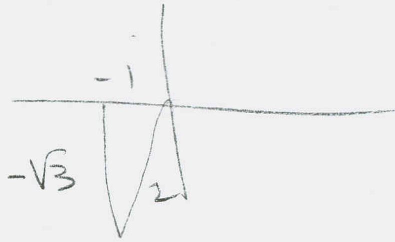
$$= \left[\frac{3}{4}\theta + \sin\theta + \frac{1}{2}\cdot\frac{1}{2}\sin(2\theta)\right]_0^{\frac{2\pi}{3}} - \left[\frac{3}{4}\theta + \sin\theta + \frac{1}{4}\sin(2\theta)\right]_{\frac{2\pi}{3}}^{\pi}$$

$$= \frac{3}{4}\cdot\frac{2\pi}{3} + \sin\left(\frac{2\pi}{3}\right) + \frac{1}{4}\sin\left(\frac{4\pi}{3}\right) - (0+0+0)$$

$$- \left[\frac{3}{4}\cdot\pi + \sin(\pi) + \frac{1}{4}\sin(2\pi) - \left(\frac{3}{4}\cdot\frac{2\pi}{3} + \sin\left(\frac{2\pi}{3}\right) + \frac{\sin\left(\frac{4\pi}{3}\right)}{4}\right)\right]$$

$$= \frac{\pi}{2} + \sin\left(\frac{2\pi}{3}\right) + \frac{1}{4}\sin\left(\frac{4\pi}{3}\right) - \frac{3\pi}{4} - 0 - 0 + \frac{\pi}{2} + \sin\left(\frac{2\pi}{3}\right) + \frac{1}{4}\sin\left(\frac{4\pi}{3}\right)$$

$$= \frac{\pi}{4} + 2\sin\left(\frac{2\pi}{3}\right) + \frac{1}{2}\sin\left(\frac{4\pi}{3}\right)$$



$$= \frac{\pi}{4} + 2\cdot\frac{\sqrt{3}}{2} + \frac{1}{2}\cdot\left(-\frac{\sqrt{3}}{2}\right)$$

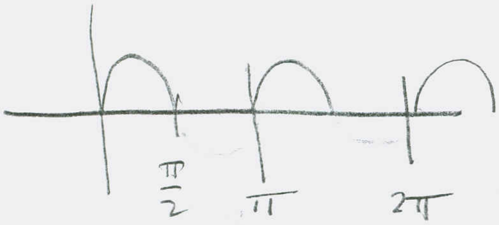
$$= \frac{\pi}{4} + \sqrt{3} - \frac{\sqrt{3}}{4} = \boxed{\frac{\pi + 3\sqrt{3}}{4}}$$

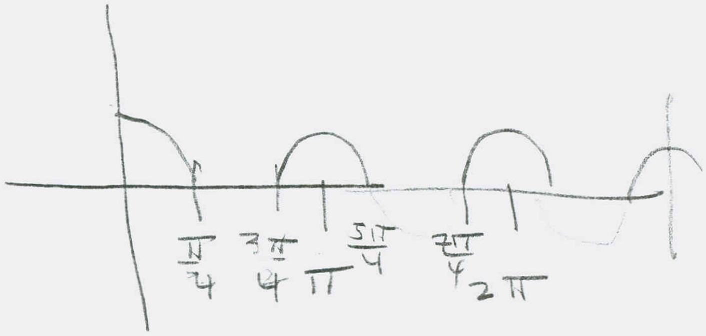
202 § 11.4 #s 39, 42, 45, 52

(42) Find all points of intersection

$r^2 = \sin(2\theta)$ $\qquad$ $r^2 = \cos(2\theta)$

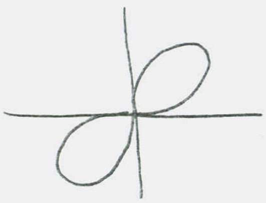
Symmetry thru the pole for both

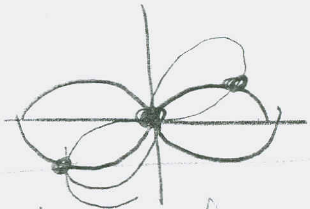




Defined for

$\theta \in [0, \frac{\pi}{2}] \cup [\pi, \frac{3\pi}{2}]$





The pole is an intersection point.

Also $r^2 = r^2$

$\sin(2\theta) = \cos(2\theta)$

$\frac{\sin(2\theta)}{\cos(2\theta)} = 1 \implies 2\theta = \frac{\pi}{4} + 2n\pi$, since

Both $\sin(2\theta)$ & $\cos(2\theta)$ must be positive.

So $\theta = \frac{\pi}{8} + n\pi$, which gives

$\theta = \frac{\pi}{8}, \frac{9\pi}{8}$

$$\boxed{\left(\frac{1}{\sqrt[4]{2}}, \frac{\pi}{8}\right) \quad \left(\frac{1}{\sqrt[4]{2}}, \frac{9\pi}{8}\right)}$$

$\theta = \frac{\pi}{8} \implies \sin(2\theta) = \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

@ $\theta = \frac{9\pi}{8}$, $\sin(2\theta) = \sin\left(\frac{9\pi}{4}\right) = \frac{1}{\sqrt{2}}$

and $r^2 = \frac{1}{\sqrt{2}} \implies r = \frac{1}{\sqrt[4]{2}} = \frac{\sqrt[4]{8}}{2}$

202 §11.4 #s 45, 52

Find the length of the polar curve

(45) $r = 3\sin\theta, \quad 0 \le \theta \le \frac{\pi}{3}$

$r^2 = 9\sin^2\theta$

Arc length in polar coords!

$\frac{dr}{d\theta} = 3\cos\theta$

$\left(\frac{dr}{d\theta}\right)^2 = 9\cos^2\theta$

$$L = \int_0^{\frac{\pi}{3}} \sqrt{9\sin^2\theta + 9\cos^2\theta}\, d\theta$$

$$= \int_0^{\frac{\pi}{3}} 3\, d\theta = 3\theta \Big]_0^{\frac{\pi}{3}} = 3\frac{\pi}{3} = \boxed{\pi}$$

(52) Same, but use calculator. 4 decimal places.

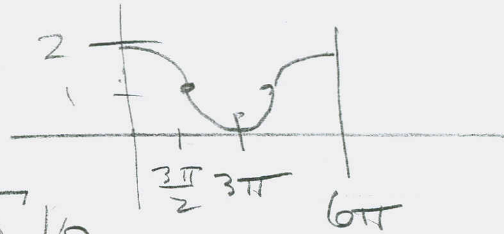
$r = 1 + \cos\left(\frac{\theta}{3}\right)$

$r^2 = 1 + 2\cos\left(\frac{\theta}{3}\right) + \cos^2\left(\frac{\theta}{3}\right) = \underline{\left(1+\cos\left(\frac{\theta}{3}\right)\right)^2}$

enter this.

$\frac{dr}{d\theta} = -\frac{1}{3}\sin\left(\frac{\theta}{3}\right)$

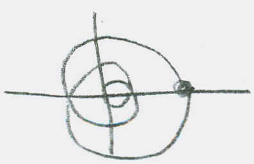
$\left(\frac{dr}{d\theta}\right)^2 = \frac{1}{9}\sin^2\left(\frac{\theta}{3}\right)$

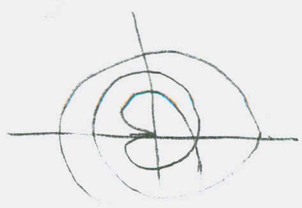


$$L = \int_0^{6\pi} \sqrt{\left(1+\cos\left(\frac{\theta}{3}\right)\right)^2 + \frac{1}{9}\sin^2\left(\frac{\theta}{3}\right)}\, d\theta$$

≈ 19.667556

$\approx \boxed{19.6676}$

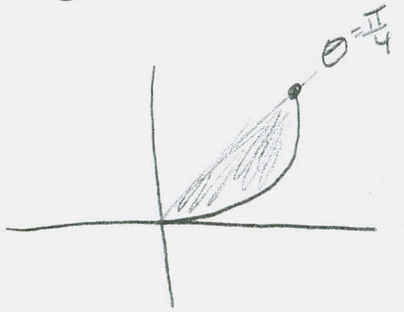




202 §11.4 #s 1, 4, 7, 10, 13, 15, 26, 31, 35, 39, 42, 45, 52,

#s 1-4 Find area of region described.

(1) $r = \theta^2,\ 0 \le \theta \le \pi/4$

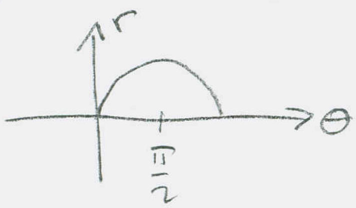


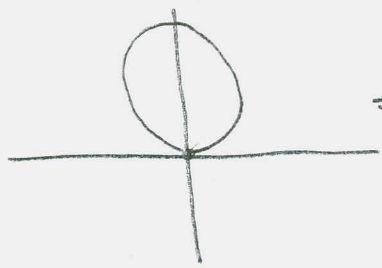
$$A = \frac{1}{2}\int_a^b f(\theta)^2\, d\theta$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{4}} \theta^4\, d\theta = \frac{1}{10}\theta^5\Big]_0^{\frac{\pi}{4}} = \frac{1}{10}\cdot\frac{\pi^5}{4^5}$$

$$= \frac{1}{10240}\pi^5$$

(4) $r = \sqrt{\sin\theta},\ 0 \le \theta \le \pi$



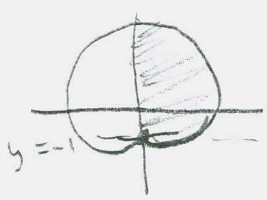


$$A = \frac{1}{2}\int_0^{\pi} \sin\theta\, d\theta$$

$$= \frac{1}{2}\cdot 2\int_0^{\frac{\pi}{2}} \cos\theta\, d\theta$$

$$= \sin\theta\Big]_0^{\frac{\pi}{2}} = 1$$

(7) Find area of shaded region



$$\frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (4 + 3\sin\theta)^2\, d\theta$$

$$= \frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (16 + 24\sin\theta + 9\sin^2\theta)\, d\theta$$

$r = 4 + 3\sin\theta > 0\ \forall x$

$$= \frac{1}{2}\int_{-\pi/2}^{\frac{\pi}{2}} 16 + 24\sin\theta + \frac{9}{2}(1 - \cos(2\theta))\, d\theta$$