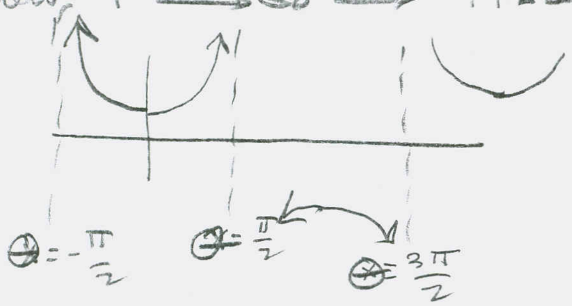
202 § 11.3 II #s 51, 55, 57, 60, 63, 66, 69, 71, 73, 77, 78

(51) Show that the polar curve $r = 4 + 2\sec\theta$ (conchoid) has the line $x = 2$ as a vertical asymptote, by showing that $\lim_{r\to\pm\infty} x = 2$

$x = r\cos\theta = (4 + 2\sec\theta)\cos\theta = 4\cos\theta + 2$.

Now, $r \longrightarrow \infty \Longrightarrow 4 + 2\sec\theta \longrightarrow \infty$ and that happens

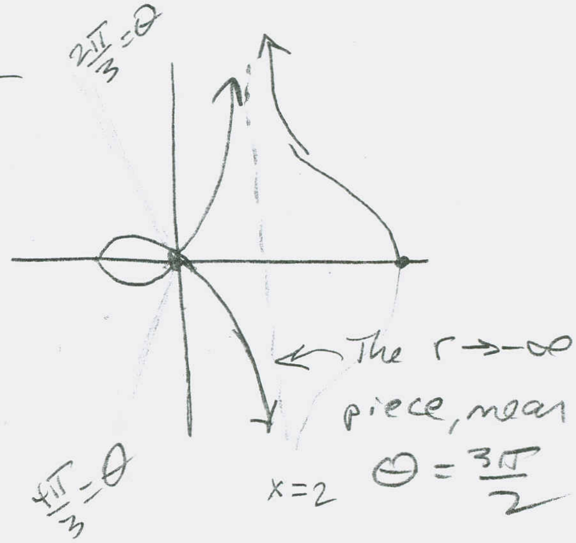


when $\theta \to \frac{\pi}{2}^-$ and/or
when $\theta \to -\frac{\pi}{2}^+$ and/or
when $\theta \to \frac{3\pi}{2}^+$, which is better since that's how we graph these ($[0, 2\pi]$)

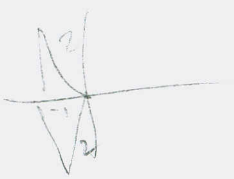
So, $\lim_{r\to\infty} x = \lim_{\theta\to\frac{\pi}{2}^-} (4\cos\theta + 2) = 2$

What's happening as $\theta \to \frac{\pi}{2}^+$?

$r \longrightarrow -\infty$



$r = 4 + 2\sec\theta \overset{\text{SET}}{=} 0$



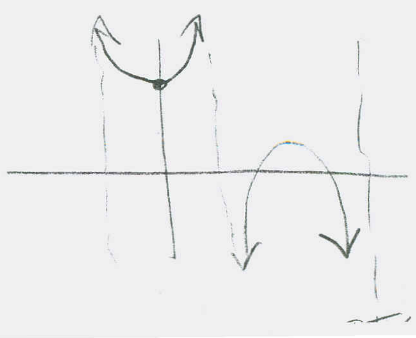
$2\sec\theta = -4$

$\sec\theta = -2$

$\cos\theta = -\frac{1}{2}$

$\theta = \frac{2\pi}{3}$

$\theta = \frac{4\pi}{3}$



202, §11.3 II

(55) $r = 1 + c\sin\theta$ is the limaçon

What values of c result in an inner loop and what values result in a cardioid?

- $|c| > 1$ results in an inner loop, since

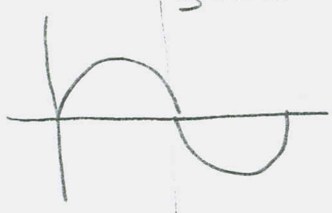
$1 + c\sin\theta = 0 \Longrightarrow$

$c\sin\theta = -1 \Longrightarrow$

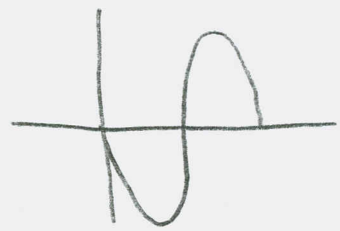
$\sin\theta = -\frac{1}{c}$ only has sol'n

$c < -1$:

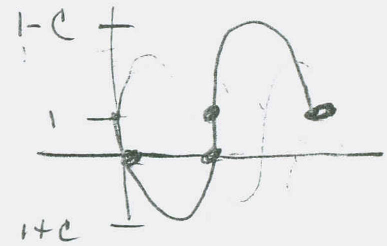
$\sin\theta$



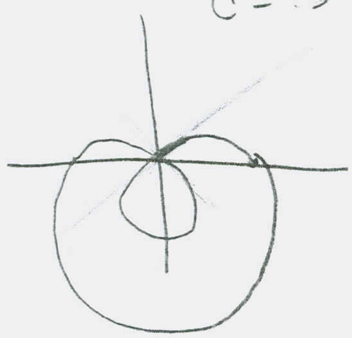
$c\sin\theta$



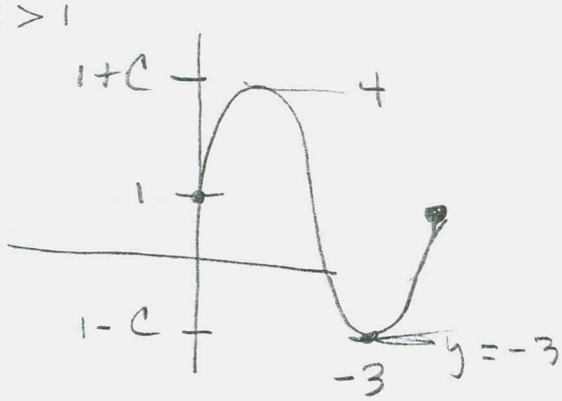
$1 + c\sin\theta$



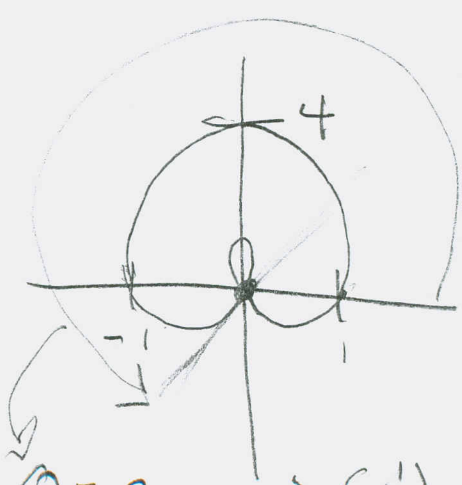
$c = -3$



$c > 1$



$c = +3$



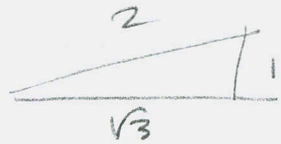
$\theta = \theta = \arcsin(-\frac{1}{3})$

202 11.3 II

57 Find the slope @ the point specified by θ.

$r = 2\sin\theta, \quad \theta = \frac{\pi}{6}$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} =$$



$$y = r\sin\theta = 2\sin\theta\sin\theta = 2\sin^2\theta \Longrightarrow$$

$$\frac{dy}{d\theta} = 4\sin\theta\cos\theta \Longrightarrow \left.\frac{dy}{d\theta}\right|_{\theta=\frac{\pi}{6}} = 4\sin\frac{\pi}{6}\cos\frac{\pi}{6} = 4\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$$

$$x = r\cos\theta = 2\sin\theta\cos\theta \Longrightarrow$$

$$\frac{dx}{d\theta} = 2\cos^2\theta - 2\sin^2\theta \Longrightarrow \left.\frac{dx}{d\theta}\right|_{\theta=\frac{\pi}{6}} = 2\left(\frac{\sqrt{3}}{2}\right)^2 - 2\left(\frac{1}{2}\right)^2$$

$$= 2\left(\frac{3}{4}\right) - 2\left(\frac{1}{4}\right)$$

$$= \frac{6-2}{4} = \frac{4}{4} = 1$$

So $\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{6}} = \frac{\sqrt{3}}{1} = \boxed{\sqrt{3}}$

Could also use book method:

$$\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{4\sin\theta\cos\theta}{2\cos^2\theta - 2\sin^2\theta} = \frac{2\sin(2\theta)}{2\cos(2\theta)} = \tan(2\theta)$$

and $\tan\left(2\cdot\frac{\pi}{6}\right) = \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$

202 §11.3 II

(60) Same as 57, but $r=\cos\left(\frac{\theta}{3}\right)$, $\theta=\pi$

$$x = r\cos\theta = \cos\left(\tfrac{\theta}{3}\right)\cos\theta$$

$$\frac{dx}{d\theta} = -\tfrac{1}{3}\sin\left(\tfrac{\theta}{3}\right)\cos\theta + \cos\left(\tfrac{\theta}{3}\right)(-\sin\theta)$$

3 pts

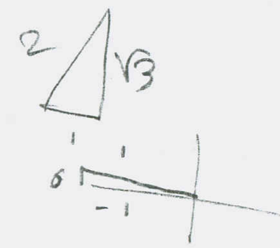
$$y = r\sin\theta = \cos\left(\tfrac{\theta}{3}\right)\sin\theta$$

#66

$$\frac{dy}{d\theta} = -\tfrac{1}{3}\sin\left(\tfrac{\theta}{3}\right)\sin\theta + \cos\left(\tfrac{\theta}{3}\right)\cos\theta$$

so

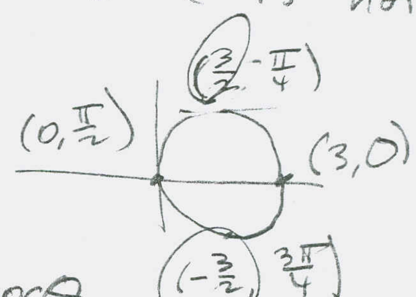
$$\left.\frac{dy}{dx}\right|_{\theta=\pi} = \frac{-\frac{1}{3}\sin\left(\frac{\pi}{3}\right)\sin\pi + \cos\left(\frac{\pi}{3}\right)\cos\pi}{-\frac{1}{3}\sin\left(\frac{\pi}{3}\right)\cos\pi - \cos\left(\frac{\pi}{3}\right)\sin\pi}$$



$$= \frac{-\frac{1}{3}\cdot\frac{\sqrt{3}}{2}\cdot 0 + \frac{1}{2}(-1)}{-\frac{1}{3}\cdot\frac{\sqrt{3}}{2}\cdot(-1) - \frac{1}{2}(0)} = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{6}} = -\frac{1}{2}\cdot\frac{6}{\sqrt{3}} = -\frac{3}{\sqrt{3}} = -\frac{3\sqrt{3}}{3} = \boxed{-\sqrt{3}}$$

#s 63-68 Find where tangent line is horizontal or vertical.

(63) $r = 3\cos\theta$



From sketching skills.

$$x = 3\cos\theta\cos\theta = 3\cos^2\theta$$

$$\frac{dx}{d\theta} = 6\cos\theta\cdot(-\sin\theta) = -6\sin\theta\cos\theta$$
$$= -3\sin(2\theta)$$

All this is, is the x-coord. of the point in rectangular coords.

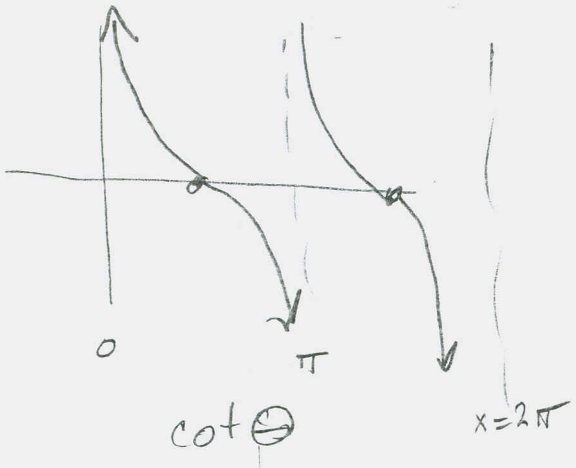
$$y = 3\cos\theta\sin\theta$$

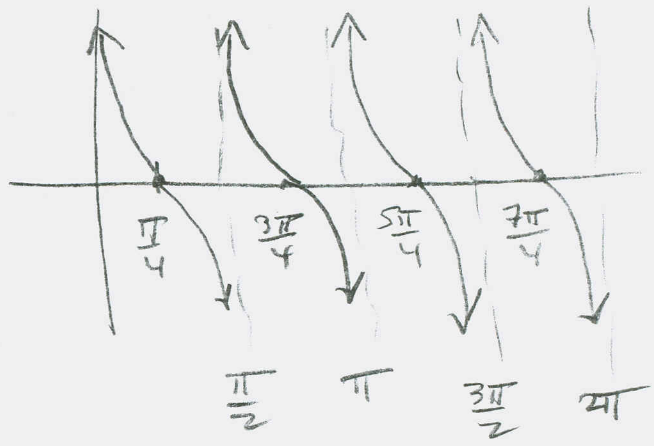
$$\frac{dy}{d\theta} = -3\sin^2\theta + 3\cos^2\theta = 3\cos(2\theta)$$

202 § 11.3 II

#63 cnt'd So $\frac{dy}{dx} = \frac{3\cos(2\theta)}{-3\sin(2\theta)} = -\cot(2\theta)$

Vertical:





$\frac{dy}{dx} = 0$ when $\theta = \frac{\pi}{4}, \frac{3\pi}{4}$

$\frac{dy}{dx}$ undefined when $\theta = 0, \frac{\pi}{2}$

$\cos 2\theta = 0$

$2\theta = \frac{\pi}{2}, \frac{3\pi}{2} \Rightarrow$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}$

$\sin(2\theta) = 0 \Rightarrow$

$2\theta = 0, \pi \Rightarrow$

$\theta = 0, \frac{\pi}{2}$

$r = 3\cos\theta$

$r\left(\frac{\pi}{4}\right) = 3\cos\left(\frac{\pi}{4}\right) = 3 \cdot \frac{1}{\sqrt{2}} = \frac{3}{\sqrt{2}}$

$\boxed{\left(\frac{3}{\sqrt{2}}, \frac{\pi}{4}\right)}$

HOR

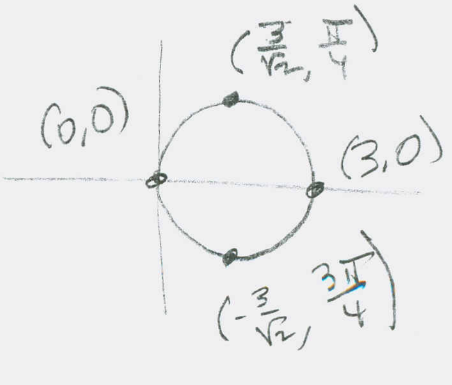
$r\left(\frac{3\pi}{4}\right) = 3\cos\left(\frac{3\pi}{4}\right) = -\frac{3}{\sqrt{2}}$

$\boxed{\left(-\frac{3}{\sqrt{2}}, \frac{3\pi}{4}\right)}$

$r(0) = 3 \;:\; (3, 0)$

$r\left(\frac{\pi}{2}\right) = 0 \;:\; (0, 0)$

VERT



202 §11.3 II #s 66, 69, 71, 73, 77, 78

(66) $r = e^{\theta}$

SPIRAL

$y = e^{\theta}\sin\theta$

$\frac{dy}{d\theta} = e^{\theta}\sin\theta + e^{\theta}\cos\theta$

$= e^{\theta}(\sin\theta + \cos\theta) \overset{SET}{=} 0$

$\Rightarrow \sin\theta = -\cos\theta$

$\Rightarrow \tan\theta = -1$

$\Rightarrow \theta = -\frac{\pi}{4} + n\pi,\ n \in \mathbb{Z}$

$\boxed{\left(e^{-\frac{\pi}{4}+n\pi},\ -\frac{\pi}{4}+n\pi\right) \text{ HOR}}$

$x = e^{\theta}\cos\theta$

$\frac{dx}{d\theta} = e^{\theta}\cos\theta - e^{\theta}\sin\theta$

$= e^{\theta}(\cos\theta - \sin\theta) \overset{SET}{=} 0$

$\Rightarrow \tan\theta = 1$

3p13

$\theta = \frac{\pi}{4} + n\pi,\ n \in \mathbb{Z}$

#78

$\boxed{\text{VERT: } \left(e^{\frac{\pi}{4}+n\pi},\ \frac{\pi}{4}+n\pi\right)}$

(69) Show that $r = a\sin\theta + b\cos\theta$, where $ab \neq 0$, represents a circle. Find its center & radius.

$x = a\sin\theta\cos\theta + b\cos^2\theta$

$y = a\sin^2\theta + b\sin\theta\cos\theta$

$x^2 = a^2\sin^2\theta\cos^2\theta + 2ab\sin\theta\cos^3\theta + b^2\cos^4\theta$

$y^2 = a^2\sin^4\theta + 2ab\sin^3\theta\cos\theta + b^2\sin^2\theta\cos^2\theta$

Don't think it leads anywhere

$(h,k) = \left(\frac{b}{2}, \frac{a}{2}\right)$

$r = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}}$

$= \frac{1}{2}\sqrt{a^2+b^2}$

$r^2 = ar\sin\theta + br\cos\theta$

$= ay + bx = x^2 + y^2 \Rightarrow$

$x^2 - bx + \left(\frac{b}{2}\right)^2 + y^2 - ay + \left(\frac{a}{2}\right)^2 = \frac{b^2}{4} + \frac{a^2}{4}$

$\left(x - \frac{b}{2}\right)^2 + \left(y - \frac{a}{2}\right)^2 = \frac{a^2}{4} + \frac{b^2}{4}$

202 §11.3 II #s 71, 73, 77, 79

#s 71–76 — Use grapher. Nephroid

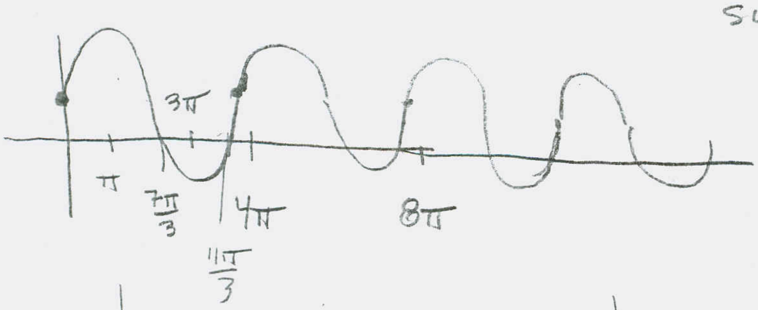
71 $r = 1 + 2\sin\left(\frac{\theta}{2}\right)$

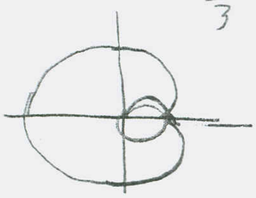
$2\sin\left(\frac{\theta}{2}\right) = -1$

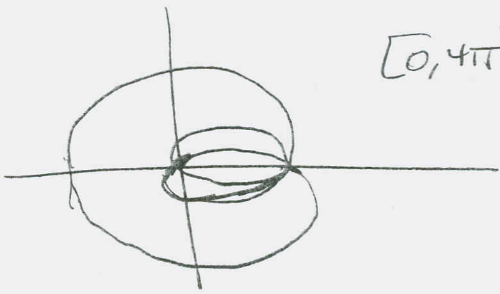
$\sin\left(\frac{\theta}{2}\right) = -\frac{1}{2}$

$\frac{\theta}{2} = \frac{7\pi}{6}, \frac{11\pi}{6}$

$\theta = \frac{7\pi}{3}, \frac{11\pi}{3}$

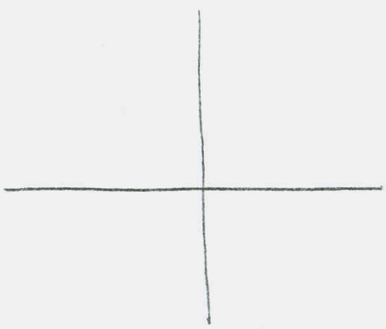






$[0, 4\pi]$

73 $r = e^{\sin\theta} - 2\cos(4\theta)$ Butterfly



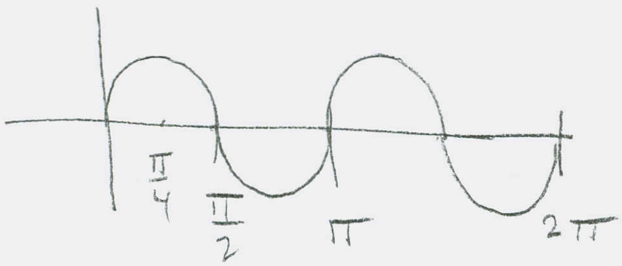
Bleah.

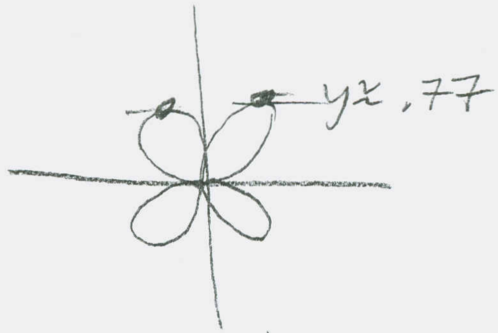
77 $r = 1 + \sin\left(\theta - \frac{\pi}{6}\right)$ and $r = 1 + \sin\left(\theta - \frac{\pi}{3}\right)$ are related to the graph of $r = 1 + \sin\theta$ how?

They're a rotation by $\frac{\pi}{6}$, $\frac{\pi}{3}$, respectively, counterclockwise. Cool.

202 § 11.3 II

78 Use a graph to estimate the high pts on $r = \sin(2\theta)$ Then use calculus





$y = \sin(2\theta)\sin\theta$

$y' = 2\cos(2\theta)\sin\theta + \sin(2\theta)\cos\theta = \frac{dy}{d\theta}$

SET = 0

$= 2(2\cos^2\theta - 1)\sin\theta + 2\sin\theta\cos\theta\cos\theta$

$= 2\sin\theta\left[2\cos^2\theta - 1 + \cos^2\theta\right]$

$= 2\sin\theta\left[3\cos^2\theta - 1\right] = 0 \Longrightarrow$

$\sin\theta = 0$

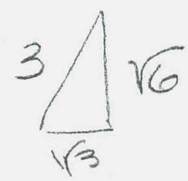
$\theta = 0, \pi$

$3\cos^2\theta = 1$

$\cos^2\theta = \frac{1}{3}$

$\cos\theta = \pm\frac{1}{\sqrt{3}} = \pm\frac{\sqrt{3}}{3}$

$\theta = \arccos\left(\pm\frac{\sqrt{3}}{3}\right)$



1st Quadrant:

$y = \sin(2\theta)\sin\theta = 2\sin^2\theta\cos\theta$

$= 2\cdot\left(\frac{\sqrt{6}}{3}\right)^2\cdot\frac{\sqrt{3}}{3} = \frac{2\cdot 6\cdot\sqrt{3}}{9\cdot 3} \approx \boxed{.7698003589}$

y-value