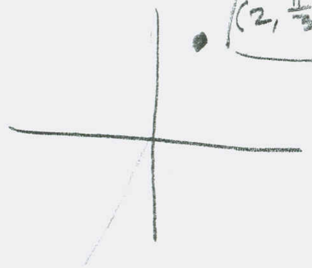
202 §11.3 I #s 1, 3, 6, 8, 9, 14, 17, 20, 23, 26, 27, 29, 32, 40, 43

① Plot the point, which is given in polar coordinates. Then find 2 other points, one with $r>0$, one with $r<0$ that are the same location

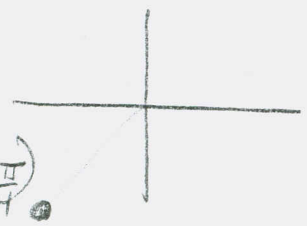
(a) $\left(2, \frac{\pi}{3}\right)$ $\qquad \pi + \frac{\pi}{3} = \frac{4\pi}{3} \qquad 2\pi + \frac{\pi}{3} = \frac{7\pi}{3}$

$$\boxed{\left(2, \frac{\pi}{3}\right) = \left(-2, \frac{4\pi}{3}\right) = \left(2, \frac{7\pi}{3}\right)}$$



(1b) $\left(1, -\frac{3\pi}{4}\right)$ $\qquad -\frac{3\pi}{4} + \pi = \frac{\pi}{4}$

$-\frac{3\pi}{4} + 2\pi = \frac{5\pi}{4}$



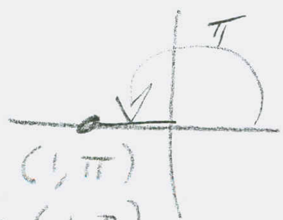
$\left(1, -\frac{3\pi}{4}\right)$

$= \left(-1, \frac{\pi}{4}\right) = \left(1, \frac{5\pi}{4}\right)$

③ Find the point whose polar coordinates are given. Then find its cartesian representation.

(a) $(1, \pi)$ $\qquad x = 1 \cos \pi = -1, \; y = 1 \sin \pi = 0$

$\longrightarrow (-1, 0)$



$(1, \pi)$

$= (-1, 0)$

202 §11.3 I #s 6, 8, 9, 14, 17, 20, 23, 26, 27, 29, 32, 40, 43

(6) The Cartesian are given:

(i) Find polar coordinates where $r>0$ & $0 \le \theta < 2\pi$

(ii) " " " " $r<0$ & $0 \le \theta < 2\pi$

(a) $(3\sqrt{3}, 3)$

$x^2+y^2=r^2$

$(3\sqrt{3})^2+(3)^2=r^2$

$9 \cdot 3 + 9 = r^2$

$27+9=r^2$

$36=r^2$

$6=r$

$x = r\cos\theta$

$3\sqrt{3} = 6\cos\theta$

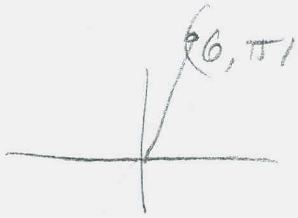
$\frac{\sqrt{3}}{2} = \cos\theta$

$\frac{\pi}{3} = \theta$

(i) $(r,\theta) = \boxed{(6, \frac{\pi}{3})}$

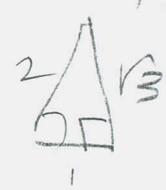
(ii) $\pi + \frac{\pi}{3} = \frac{4\pi}{3}$

$\boxed{(-6, \frac{4\pi}{3})}$



$\frac{y}{x} = \tan\theta$ is another way,

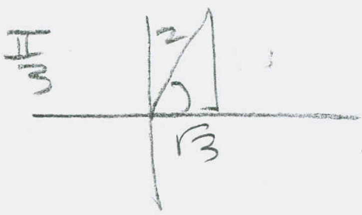
$\frac{3}{3\sqrt{3}} = \tan^{-1}\theta = \frac{1}{\sqrt{3}}$



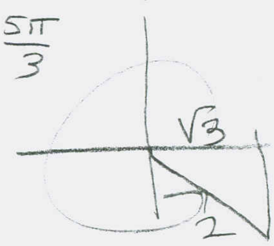
$\theta = \frac{\pi}{3}$

Thing is, $\cos\theta = \frac{\sqrt{3}}{2}$ happens twice:

$\theta = \frac{\pi}{3}$



$\theta = \frac{5\pi}{3}$



b) $(1,-2)$

$x^2+y^2=r^2$

$1+4=r^2$

$\sqrt{5}=r$

$1 = \sqrt{5}\cos\theta$

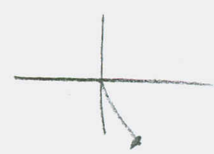
$\frac{1}{\sqrt{5}} = \cos\theta$

$\theta = \arccos\left(\frac{1}{\sqrt{5}}\right)$

of course, you can (should) check:

$-2 = \sqrt{5}\sin\theta$

$-\frac{2}{\sqrt{5}} = \sin\theta \longrightarrow \theta = \arcsin\left(-\frac{2}{\sqrt{5}}\right)$



202 §11.3 #s 6, 8, 9, 14, 17, 20, 23, 26, 27, 29, 32, 40, 43

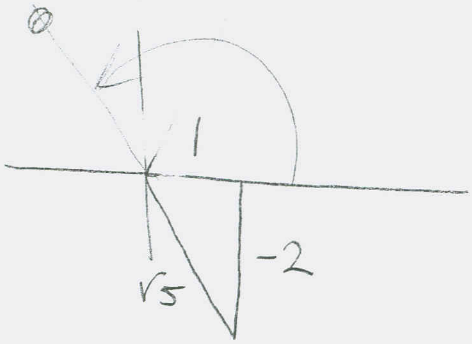
(6) cnt'd.

Here's the thing: arcsine has range between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$. We want our values of θ between 0 and 2π.

__BUT__, $\arcsin(-\frac{2}{\sqrt{5}})$ will be between $-\frac{\pi}{2}$ and 0,

Also, arccosine is defined for $0 \leq \theta < \pi$

And $\arccos(\frac{1}{\sqrt{5}})$ will be between 0 and $\frac{\pi}{2}$;



even though we know our θ __MUST__ be between $\frac{3\pi}{2}$ and 2π, since (x,y) is in the 4th quadrant.

That's why the book just uses arctangent, because it's easier to rectify:

(i) $(\sqrt{5}, \arctan(-2)+2\pi)$ OR $(\sqrt{5}, 2\pi - \arctan(2))$

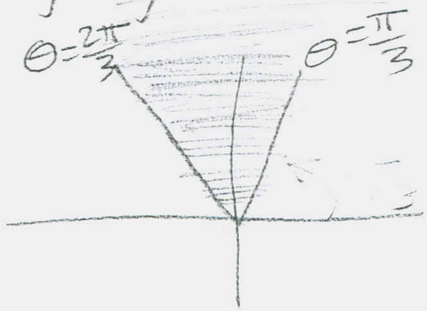
since arctangent is odd.

(ii) $(-\sqrt{5}, \arctan(-2)+\pi)$ OR $(-\sqrt{5}, \pi - \arctan(2))$

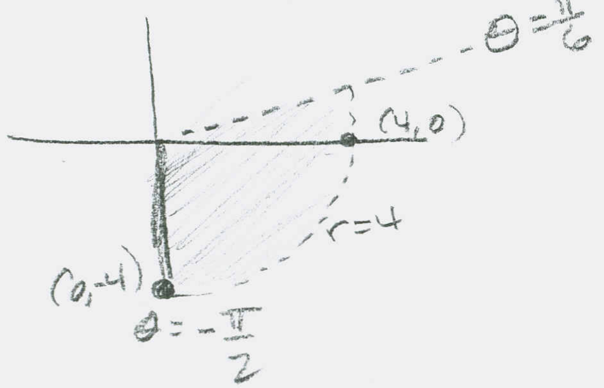
202 § 11.3 #s 8, 9, 14, 17, 20, 23, 26, 27, 29, 32, 40, 43

(8) Sketch the region satisfying the conditions

$r > 0, \ \frac{\pi}{3} \le \theta \le \frac{2\pi}{3}$



(9) Same. $0 \le r < 4$ $\quad -\frac{\pi}{2} \le \theta < \frac{\pi}{6}$



(14) Find formula for distance between points with polar coordinates (r_1, θ_1) & (r_2, θ_2)

$x = r\cos\theta, \ y = r\sin\theta$

$$D^2 = (r_1\cos\theta_1 - r_2\cos\theta_2)^2 + (r_1\sin\theta_1 - r_2\sin\theta_2)^2$$

$$= r_1^2\cos^2\theta_1 - 2r_1r_2\cos\theta_1\cos\theta_2 + r_2^2\cos^2\theta_2$$

$$+ r_1^2\sin^2\theta_1 - 2r_1r_2\sin\theta_1\sin\theta_2 + r_2^2\sin^2\theta_2$$

$$= r_1^2 - 2r_1r_2(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2) + r_2^2$$

$$= r_1^2 - 2r_1r_2(\cos(\theta_1 - \theta_2)) + r_2^2$$

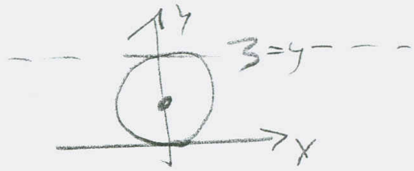
so $D = \sqrt{\text{above}}$.

201 §11.3 #s 17, 20, 23, 26, 27, 29, 32, 40, 43

#s 17–20 Identify the curve by converting to cartesian coords.

17 $r = 3\sin\theta$ Using $y = r\sin\theta$:

$r^2 = 9\sin^2\theta = 3r\sin\theta = 3y$



But $r^2 = x^2 + y^2 = 3y$, so

$x^2 + y^2 - 3y = 0$

$x^2 + y^2 - 3y + \left(\frac{3}{2}\right)^2 = \left(\frac{3}{2}\right)^2$

$x^2 + \left(y - \frac{3}{2}\right)^2 = \left(\frac{3}{2}\right)^2$

circle, Radius $\left(\frac{3}{2}\right)$ centered @ $\left(0, \frac{3}{2}\right)$

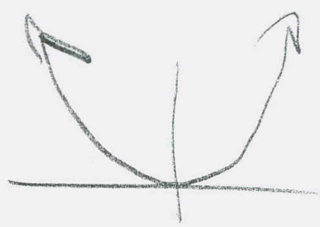
20 $r = \tan\theta\sec\theta = \dfrac{\sin\theta}{\cos^2\theta} \Longrightarrow$

#32

$r\cos^2\theta = \sin\theta \Longrightarrow$

$r^2\cos^2\theta = r\sin\theta \Longrightarrow$

$\boxed{x^2 = y}$



#s 21–26 Find a polar eq'n:

23

$x = -y^2$

$r\cos\theta = -(r\sin\theta)^2 \Longrightarrow \cos\theta = -r\sin^2\theta$

$\Longrightarrow r = -\dfrac{\cos\theta}{\sin^2\theta}$ OR $\boxed{r = -\cot\theta\csc\theta}$

202 § 11.3 I #s 26, 27, 29, 32, 40, 43

(26) $xy = 4$

$r\cos\theta \, r\sin\theta = 4$

$r^2\left(\frac{1}{2} \cdot 2\sin\theta\cos\theta\right) = 4$

$\frac{1}{2} r^2 \left(\sin(2\theta)\right) = 4$

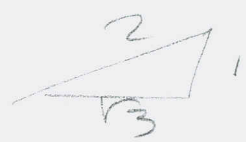
$r^2 \sin(2\theta) = 8$

$r^2 = 8\csc(2\theta)$

(27) Which is easier: rectangular or polar?

(a) A line thru the origin, making an angle of $\frac{\pi}{6}$ with the positive x-axis.

$\theta = \frac{\pi}{6}$ is simple.



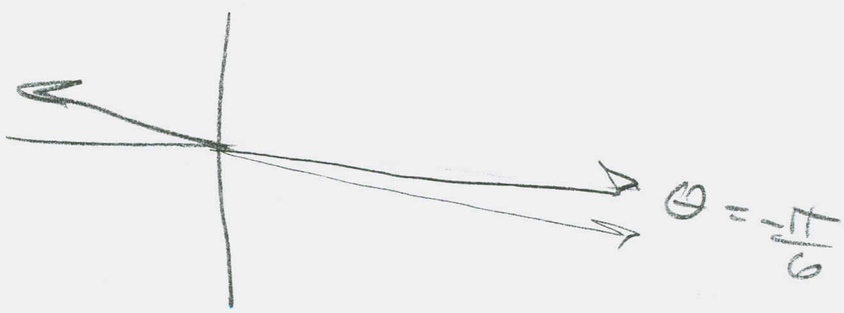
$y = \tan\left(\frac{\pi}{6}\right) x = \frac{1}{\sqrt{3}} x$ isn't bad.

(b) Vertical line thru $(3,3)$:

$\boxed{x = 3}$ is all you need.

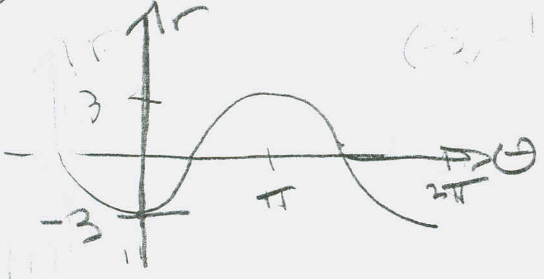
#s 29-48 Sketch the curve:

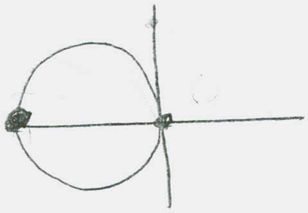
(29) $\theta = -\frac{\pi}{6}$



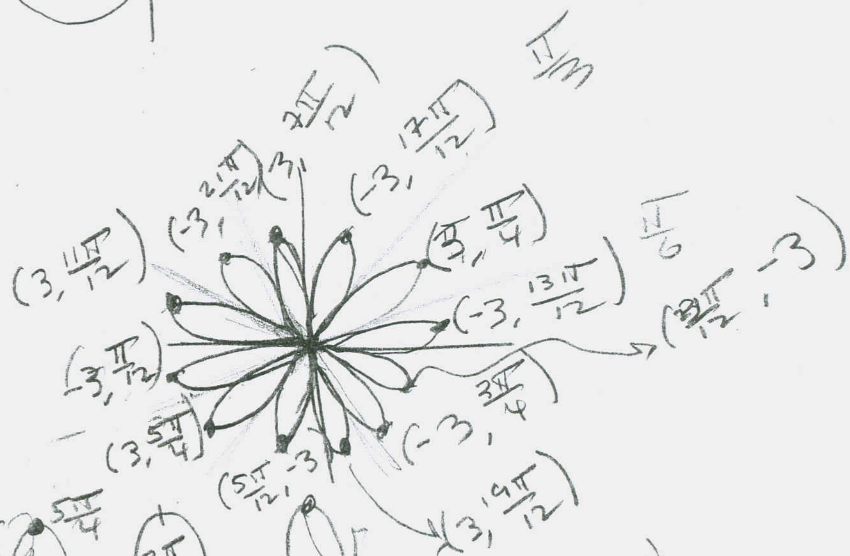
202 11.3 I #s 32, 40, 43

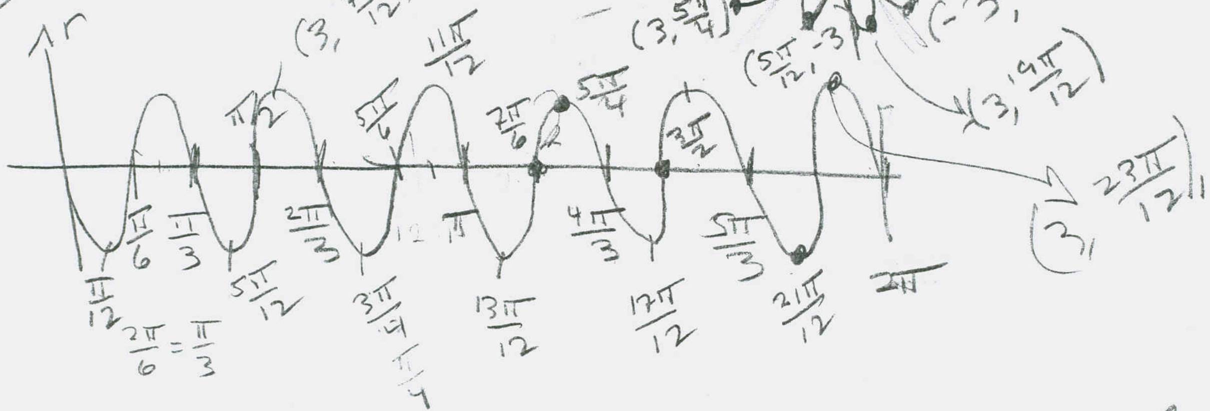
(32) $r = -3\cos\theta$





(40) $r = -3\sin(6\theta)$





12-petal rose for $r = -3\sin(6\theta)$

Unfortunately, we were doing $r = -3\cos(6\theta)$

But I basically saw that each loop was over an angle span of $\frac{\pi}{6}$. I wasn't sure, right away if I'd have 12 or 6 or ... loops, until I carefully worked my way around from $\theta = 0$ to $\theta = 2\pi$.

Can you think of tricks that will shorten the way?

202 §11.3 I #40

$r = 3\cos(6\theta) = 3\cos(6(-\theta))$ Symmetry about

sym polar axis.

$3\cos(6(\theta+\pi)) = 3\cos(6\theta + 6\pi) =$

$3\cos(6\theta)\cos(6\pi) - 3\sin(6\theta)\sin(6\pi) = 3\cos(6\theta)$

symm. thru pole.

$3\cos(6(\pi-\theta)) = 3\cos 6\pi \cos 6\theta + 3\sin 6\pi \sin 6\theta$

$= 3\cos 6\theta$

sym about $\theta = \frac{\pi}{2}$