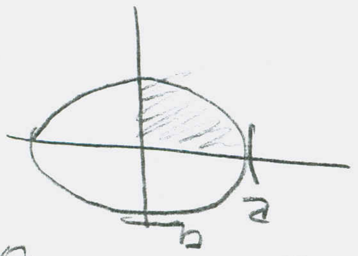
202 §11.2 II

#31 Use $x = a\cos\theta,\ y = b\sin\theta,\ 0 \le \theta \le 2\pi$, to find the area of the ellipse



$$A = 4\int_0^a y\,dx = 4\int_{\frac{\pi}{2}}^{0} f(\theta)g'(\theta)\,d\theta$$

$x = a\cos\theta$
$x' = -a\sin\theta$

$x=0 \to \theta = \frac{\pi}{2}$
$x=a \to \theta = 0$

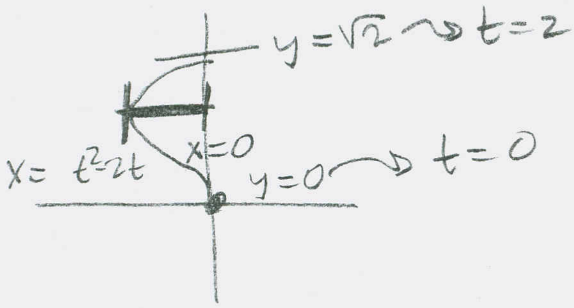
$$= 4\int_{\frac{\pi}{2}}^{0} b\sin\theta(-a\sin\theta)\,d\theta$$

$$= 4\int_0^{\frac{\pi}{2}} ab\sin^2\theta\,d\theta = 4ab\int_0^{\frac{\pi}{2}} \frac{1-\cos(2\theta)}{2}\,d\theta$$

$$= 2ab\int_0^{\frac{\pi}{2}} (1-\cos(2\theta))\,d\theta = 2ab\left[\theta - \frac{1}{2}\sin(2\theta)\right]_0^{\frac{\pi}{2}}$$

$$= 2ab\left[\frac{\pi}{2} - \frac{1}{2}\sin(\pi) - \left(0 - \frac{1}{2}\sin(0)\right)\right] = 2ab\cdot\frac{\pi}{2} = \boxed{\pi ab}$$

(32) $x = t^2 - 2t,\ y = \sqrt{t} = t^{\frac{1}{2}}$ Find area enclosed by this and the y-axis.



$$\text{Area} = \int_{y=0}^{y=\sqrt{2}} (0 - x)\,dy$$

$$= \int_{t=0}^{t=2} -(t^2-2t)\,\frac{1}{2}t^{-\frac{1}{2}}\,dt =$$

$$= -\int_0^2 \left(\frac{1}{2}t^{\frac{3}{2}} - t^{\frac{1}{2}}\right)dt = -\left[\frac{1}{5}t^{\frac{5}{2}} - \frac{2}{3}t^{\frac{3}{2}}\right]_0^2 = -\left[\frac{1}{5}(2)^{\frac{5}{2}} - \frac{2}{3}(2)^{\frac{3}{2}}\right]$$

$$= -\frac{1}{5}(2^5)^{\frac{1}{2}} + \frac{2}{3}(2^3)^{\frac{1}{2}} = -\frac{4}{5}\sqrt{2} + \frac{4}{3}\sqrt{2} = \boxed{\frac{8}{15}\sqrt{2}}$$

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(44) Find the length

$x = 3\cos(t) - \cos(3t),\ y = 3\sin(t) - \sin(3t),\ 0 \le t \le \pi$

$\frac{dx}{dt} = -3\sin t + 3\sin(3t)$

$\left(\frac{dx}{dt}\right)^2 = 9\sin^2 t - 18\sin(t)\sin(3t) + 9\sin^2(3t)$

$\frac{dy}{dt} = 3\cos(t) - 3\cos(3t)$

$\left(\frac{dy}{dt}\right)^2 = 9\cos^2(t) - 18\cos(t)\cos(3t) + 9\cos^2(3t)$

$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = 9\sin^2(t) - 18\sin(t)\sin(3t) + 9\sin^2(3t)$
$\quad + 9\cos^2(t) - 18\cos(t)\cos(3t) + 9\cos^2(3t)$

$= 9(\sin^2(t) + \cos^2(t)) + 18[\sin(t)\sin(3t) + \cos(t)\cos(3t)]$
$+ 9[\sin^2(3t) + \cos^2(3t)]$

Now, $\cos(x-y) = \cos x\cos y + \sin x \sin y \Longrightarrow$

$9(1) - 18(\cos(2t)) + 9(1)$

$= 18 - 18\cos(2t) = 18(1 - \cos(2t)) = 36\left(\frac{1-\cos(2t)}{2}\right)$

$= 36\sin^2 t \Longrightarrow$ Arc length is

$\int_0^{\pi} \sqrt{36\sin^2 t}\, dt = 6\int_0^{\pi} \sin t\, dt = 6\left[-\cos(t)\right]_0^{\pi} = 6[-(-1) - (-1)]$

$= \boxed{12}$