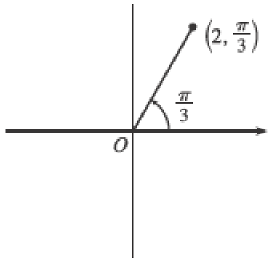
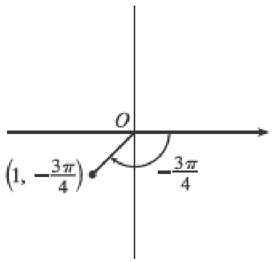
1. (a) $\left(2, \frac{\pi}{3}\right)$



By adding 2π to $\frac{\pi}{3}$, we obtain the point $\left(2, \frac{7\pi}{3}\right)$. The direction opposite $\frac{\pi}{3}$ is $\frac{4\pi}{3}$, so $\left(-2, \frac{4\pi}{3}\right)$ is a point that satisfies the $r < 0$ requirement.

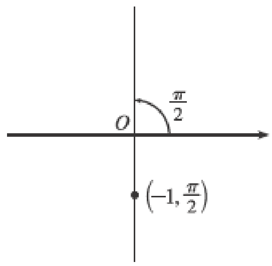
(b) $\left(1, -\frac{3\pi}{4}\right)$



$r > 0$: $\left(1, -\frac{3\pi}{4} + 2\pi\right) = \left(1, \frac{5\pi}{4}\right)$

$r < 0$: $\left(-1, -\frac{3\pi}{4} + \pi\right) = \left(-1, \frac{\pi}{4}\right)$

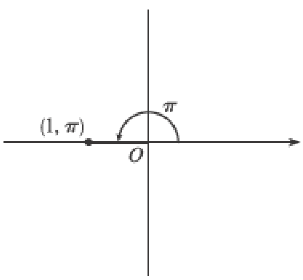
(c) $\left(-1, \frac{\pi}{2}\right)$



$r > 0$: $\left(-(-1), \frac{\pi}{2} + \pi\right) = \left(1, \frac{3\pi}{2}\right)$

$r < 0$: $\left(-1, \frac{\pi}{2} + 2\pi\right) = \left(-1, \frac{5\pi}{2}\right)$

3. (a)

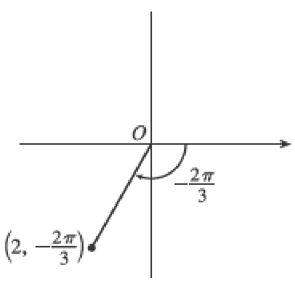


$x = 1\cos\pi = 1(-1) = -1$ and

$y = 1\sin\pi = 1(0) = 0$ give us

the Cartesian coordinates $(-1, 0)$.

(b)

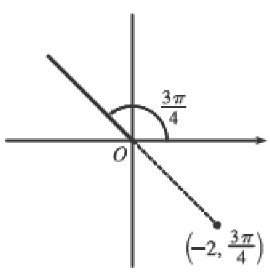


$x = 2\cos\left(-\frac{2\pi}{3}\right) = 2\left(-\frac{1}{2}\right) = -1$ and

$y = 2\sin\left(-\frac{2\pi}{3}\right) = 2\left(-\frac{\sqrt{3}}{2}\right) = -\sqrt{3}$

give us $\left(-1, -\sqrt{3}\right)$.

(c)



$x = -2\cos\frac{3\pi}{4} = -2\left(-\frac{\sqrt{2}}{2}\right) = \sqrt{2}$ and

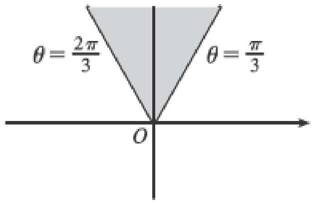
$y = -2\sin\frac{3\pi}{4} = -2\left(\frac{\sqrt{2}}{2}\right) = -\sqrt{2}$

gives us $\left(\sqrt{2}, -\sqrt{2}\right)$.

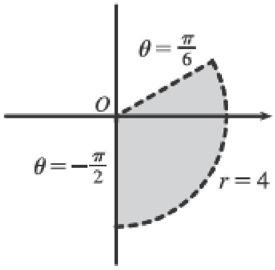
6. (a) $x = 3\sqrt{3}$ and $y = 3 \Rightarrow r = \sqrt{(3\sqrt{3})^2 + 3^2} = \sqrt{27+9} = 6$ and $\theta = \tan^{-1}\left(\frac{3}{3\sqrt{3}}\right) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$. Since $(3\sqrt{3}, 3)$ is in the first quadrant, the polar coordinates are (i) $\left(6, \frac{\pi}{6}\right)$ and (ii) $\left(-6, \frac{7\pi}{6}\right)$.

(b) $x = 1$ and $y = -2 \Rightarrow r = \sqrt{1^2 + (-2)^2} = \sqrt{5}$ and $\theta = \tan^{-1}\left(\frac{-2}{1}\right) = -\tan^{-1} 2$. Since $(1, -2)$ is in the fourth quadrant, the polar coordinates are (i) $\left(\sqrt{5}, 2\pi - \tan^{-1} 2\right)$ and (ii) $\left(-\sqrt{5}, \pi - \tan^{-1} 2\right)$.

8. $r \geq 0, \quad \pi/3 \leq \theta \leq 2\pi/3$



9. The region satisfying $0 \leq r < 4$ and $-\pi/2 \leq \theta < \pi/6$ does not include the circle $r = 4$ nor the line $\theta = \frac{\pi}{6}$.



14. The points (r_1, θ_1) and (r_2, θ_2) in Cartesian coordinates are $(r_1 \cos\theta_1, r_1 \sin\theta_1)$ and $(r_2 \cos\theta_2, r_2 \sin\theta_2)$, respectively. The *square* of the distance between them is

$$\begin{aligned}
&(r_2\cos\theta_2 - r_1\cos\theta_1)^2 + (r_2\sin\theta_2 - r_1\sin\theta_1)^2 \\
&\qquad = \left(r_2^2\cos^2\theta_2 - 2r_1r_2\cos\theta_1\cos\theta_2 + r_1^2\cos^2\theta_1\right) + \left(r_2^2\sin^2\theta_2 - 2r_1r_2\sin\theta_1\sin\theta_2 + r_1^2\sin^2\theta_1\right) \\
&\qquad = r_1^2\left(\sin^2\theta_1 + \cos^2\theta_1\right) + r_2^2\left(\sin^2\theta_2 + \cos^2\theta_2\right) - 2r_1r_2(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2) \\
&\qquad = r_1^2 - 2r_1r_2\cos(\theta_1 - \theta_2) + r_2^2,
\end{aligned}$$

so the distance between them is $\sqrt{r_1^2 - 2r_1r_2\cos(\theta_1 - \theta_2) + r_2^2}$.

17. $r = 3\sin\theta \Rightarrow r^2 = 3r\sin\theta \Leftrightarrow x^2 + y^2 = 3y \Leftrightarrow x^2 + \left(y - \frac{3}{2}\right)^2 = \left(\frac{3}{2}\right)^2$, a circle of radius $\frac{3}{2}$ centered at $\left(0, \frac{3}{2}\right)$. The first two equations are actually equivalent since $r^2 = 3r\sin\theta \Rightarrow r(r - 3\sin\theta) = 0 \Rightarrow r = 0$ or $r = 3\sin\theta$. But $r = 3\sin\theta$ gives the point $r = 0$ (the pole) when $\theta = 0$. Thus, the single equation $r = 3\sin\theta$ is equivalent to the compound condition ($r = 0$ or $r = 3\sin\theta$).

20. $r = \tan\theta\sec\theta = \dfrac{\sin\theta}{\cos^2\theta} \Rightarrow r\cos^2\theta = \sin\theta \Leftrightarrow (r\cos\theta)^2 = r\sin\theta \Leftrightarrow x^2 = y$, a parabola with vertex at the origin opening upward. The first implication is reversible since $\cos\theta = 0$ would imply $\sin\theta = r\cos^2\theta = 0$, contradicting the fact that $\cos^2\theta + \sin^2\theta = 1$.

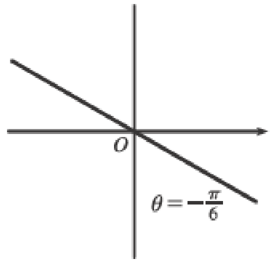
23. $x = -y^2 \Leftrightarrow r\cos\theta = -r^2\sin^2\theta \Leftrightarrow \cos\theta = -r\sin^2\theta \Leftrightarrow r = -\dfrac{\cos\theta}{\sin^2\theta} = -\cot\theta\csc\theta$.

26. $xy = 4 \Leftrightarrow (r\cos\theta)(r\sin\theta) = 4 \Leftrightarrow r^2\left(\frac{1}{2}\cdot 2\sin\theta\cos\theta\right) = 4 \Leftrightarrow r^2\sin 2\theta = 8 \Rightarrow r^2 = 8\csc 2\theta$

27. (a) The description leads immediately to the polar equation $\theta = \frac{\pi}{6}$, and the Cartesian equation $y = \tan\left(\frac{\pi}{6}\right)x = \frac{1}{\sqrt{3}}x$ is slightly more difficult to derive.

(b) The easier description here is the Cartesian equation $x = 3$.

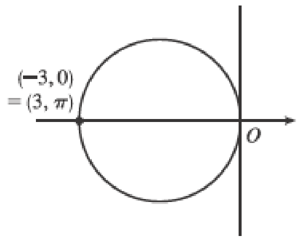
29. $\theta = -\pi/6$



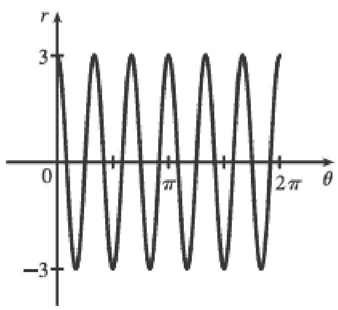
32. $r = -3\cos\theta \Leftrightarrow r^2 = -3r\cos\theta \Leftrightarrow$

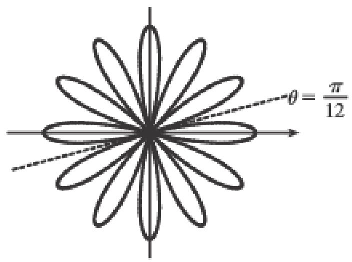
$x^2 + y^2 = -3x \Leftrightarrow \left(x + \frac{3}{2}\right)^2 + y^2 = \left(\frac{3}{2}\right)^2$.

This curve is a circle of radius $\frac{3}{2}$ centered at $\left(-\frac{3}{2}, 0\right)$.



40. $r = 3\cos 6\theta$





43. $r^2 = 9\sin 2\theta$