

31. By symmetry of the ellipse about the x - and y -axes,

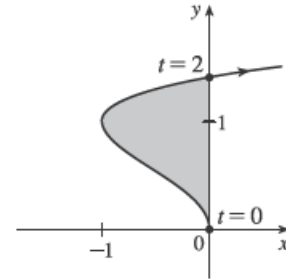
$$\begin{aligned} A &= 4 \int_0^a y \, dx = 4 \int_{\pi/2}^0 b \sin \theta (-a \sin \theta) \, d\theta = 4ab \int_0^{\pi/2} \sin^2 \theta \, d\theta = 4ab \int_0^{\pi/2} \frac{1}{2}(1 - \cos 2\theta) \, d\theta \\ &= 2ab \left[\theta - \frac{1}{2} \sin 2\theta \right]_0^{\pi/2} = 2ab \left(\frac{\pi}{2} \right) = \pi ab \end{aligned}$$

32. The curve $x = t^2 - 2t = t(t - 2)$, $y = \sqrt{t}$ intersects the y -axis when $x = 0$,

that is, when $t = 0$ and $t = 2$. The corresponding values of y are 0 and $\sqrt{2}$.

The shaded area is given by

$$\begin{aligned} \int_{y=0}^{y=\sqrt{2}} (x_R - x_L) \, dy &= \int_{t=0}^{t=2} [0 - x(t)] y'(t) \, dt = - \int_0^2 (t^2 - 2t) \left(\frac{1}{2\sqrt{t}} \right) dt \\ &= - \int_0^2 \left(\frac{1}{2} t^{3/2} - t^{1/2} \right) dt = - \left[\frac{1}{5} t^{5/2} - \frac{2}{3} t^{3/2} \right]_0^2 \\ &= - \left(\frac{1}{5} \cdot 2^{5/2} - \frac{2}{3} \cdot 2^{3/2} \right) = -2^{1/2} \left(\frac{4}{5} - \frac{4}{3} \right) = -\sqrt{2} \left(-\frac{8}{15} \right) = \frac{8}{15} \sqrt{2} \end{aligned}$$



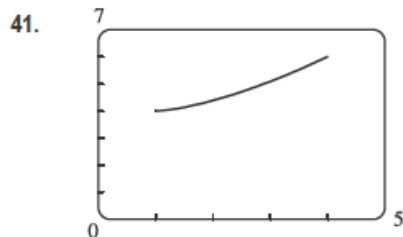
37. $x = t - t^2$, $y = \frac{4}{3}t^{3/2}$, $1 \leq t \leq 2$. $dx/dt = 1 - 2t$ and $dy/dt = 2t^{1/2}$, so

$$(dx/dt)^2 + (dy/dt)^2 = (1 - 2t)^2 + (2t^{1/2})^2 = 1 - 4t + 4t^2 + 4t = 1 + 4t^2.$$

$$\text{Thus, } L = \int_a^b \sqrt{(dx/dt)^2 + (dy/dt)^2} \, dt = \int_1^2 \sqrt{1 + 4t^2} \, dt \approx 3.1678.$$

38. $x = 1 + e^t$, $y = t^2$, $-3 \leq t \leq 3$. $dx/dt = e^t$ and $dy/dt = 2t$, so $(dx/dt)^2 + (dy/dt)^2 = e^{2t} + 4t^2$.

$$\text{Thus, } L = \int_a^b \sqrt{(dx/dt)^2 + (dy/dt)^2} \, dt = \int_{-3}^3 \sqrt{e^{2t} + 4t^2} \, dt \approx 30.5281.$$



$$x = 1 + 3t^2, \quad y = 4 + 2t^3, \quad 0 \leq t \leq 1.$$

$$dx/dt = 6t \text{ and } dy/dt = 6t^2, \text{ so } (dx/dt)^2 + (dy/dt)^2 = 36t^2 + 36t^4.$$

$$\begin{aligned} \text{Thus, } L &= \int_0^1 \sqrt{36t^2 + 36t^4} \, dt = \int_0^1 6t \sqrt{1 + t^2} \, dt \\ &= 6 \int_1^2 \sqrt{u} \left(\frac{1}{2} du \right) \quad [u = 1 + t^2, du = 2t \, dt] \\ &= 3 \left[\frac{2}{3} u^{3/2} \right]_1^2 = 2(2^{3/2} - 1) = 2(2\sqrt{2} - 1) \end{aligned}$$

44. $x = 3 \cos t - \cos 3t$, $y = 3 \sin t - \sin 3t$, $0 \leq t \leq \pi$. $\frac{dx}{dt} = -3 \sin t + 3 \sin 3t$ and $\frac{dy}{dt} = 3 \cos t - 3 \cos 3t$, so

$$\begin{aligned} \left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2 &= 9 \sin^2 t - 18 \sin t \sin 3t + 9 \sin^2(3t) + 9 \cos^2 t - 18 \cos t \cos 3t + 9 \cos^2(3t) \\ &= 9(\cos^2 t + \sin^2 t) - 18(\cos t \cos 3t + \sin t \sin 3t) + 9[\cos^2(3t) + \sin^2(3t)] \\ &= 9(1) - 18 \cos(t - 3t) + 9(1) = 18 - 18 \cos(-2t) = 18(1 - \cos 2t) \\ &= 18[1 - (1 - 2 \sin^2 t)] = 36 \sin^2 t. \end{aligned}$$

$$\text{Thus, } L = \int_0^\pi \sqrt{36 \sin^2 t} \, dt = 6 \int_0^\pi |\sin t| \, dt = 6 \int_0^\pi \sin t \, dt = -6[\cos t]_0^\pi = -6(-1 - 1) = 12.$$

49. $x = t - e^t$, $y = t + e^t$, $-6 \leq t \leq 6$.

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = (1 - e^t)^2 + (1 + e^t)^2 = (1 - 2e^t + e^{2t}) + (1 + 2e^t + e^{2t}) = 2 + 2e^{2t}, \text{ so } L = \int_{-6}^6 \sqrt{2 + 2e^{2t}} dt.$$

Set $f(t) = \sqrt{2 + 2e^{2t}}$. Then by Simpson's Rule with $n = 6$ and $\Delta t = \frac{6 - (-6)}{6} = 2$, we get

$$L \approx \frac{2}{3}[f(-6) + 4f(-4) + 2f(-2) + 4f(0) + 2f(2) + 4f(4) + f(6)] \approx 612.3053.$$

51. $x = \sin^2 t$, $y = \cos^2 t$, $0 \leq t \leq 3\pi$.

$$(dx/dt)^2 + (dy/dt)^2 = (2 \sin t \cos t)^2 + (-2 \cos t \sin t)^2 = 8 \sin^2 t \cos^2 t = 2 \sin^2 2t \Rightarrow$$

$$\text{Distance} = \int_0^{3\pi} \sqrt{2} |\sin 2t| dt = 6\sqrt{2} \int_0^{\pi/2} \sin 2t dt \quad [\text{by symmetry}] = -3\sqrt{2} [\cos 2t]_0^{\pi/2} = -3\sqrt{2}(-1 - 1) = 6\sqrt{2}.$$

The full curve is traversed as t goes from 0 to $\frac{\pi}{2}$, because the curve is the segment of $x + y = 1$ that lies in the first quadrant (since $x, y \geq 0$), and this segment is completely traversed as t goes from 0 to $\frac{\pi}{2}$. Thus, $L = \int_0^{\pi/2} \sin 2t dt = \sqrt{2}$, as above.

53. $x = a \sin \theta$, $y = b \cos \theta$, $0 \leq \theta \leq 2\pi$.

$$\begin{aligned} \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 &= (a \cos \theta)^2 + (-b \sin \theta)^2 = a^2 \cos^2 \theta + b^2 \sin^2 \theta = a^2(1 - \sin^2 \theta) + b^2 \sin^2 \theta \\ &= a^2 - (a^2 - b^2) \sin^2 \theta = a^2 - c^2 \sin^2 \theta = a^2 \left(1 - \frac{c^2}{a^2} \sin^2 \theta\right) = a^2(1 - e^2 \sin^2 \theta) \end{aligned}$$

$$\text{So } L = 4 \int_0^{\pi/2} \sqrt{a^2(1 - e^2 \sin^2 \theta)} d\theta \quad [\text{by symmetry}] = 4a \int_0^{\pi/2} \sqrt{1 - e^2 \sin^2 \theta} d\theta.$$

57. $x = 1 + te^t$, $y = (t^2 + 1)e^t$, $0 \leq t \leq 1$.

$$\begin{aligned} \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 &= (te^t + e^t)^2 + [(t^2 + 1)e^t + e^t(2t)]^2 = [e^t(t + 1)]^2 + [e^t(t^2 + 2t + 1)]^2 \\ &= e^{2t}(t + 1)^2 + e^{2t}(t + 1)^4 = e^{2t}(t + 1)^2[1 + (t + 1)^2], \quad \text{so} \end{aligned}$$

$$S = \int 2\pi y ds = \int_0^1 2\pi(t^2 + 1)e^t \sqrt{e^{2t}(t + 1)^2(t^2 + 2t + 2)} dt = \int_0^1 2\pi(t^2 + 1)e^{2t}(t + 1) \sqrt{t^2 + 2t + 2} dt \approx 103.5999$$

60. $x = 3t - t^3$, $y = 3t^2$, $0 \leq t \leq 1$. $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = (3 - 3t^2)^2 + (6t)^2 = 9(1 + 2t^2 + t^4) = [3(1 + t^2)]^2$.

$$S = \int_0^1 2\pi \cdot 3t^2 \cdot 3(1 + t^2) dt = 18\pi \int_0^1 (t^2 + t^4) dt = 18\pi \left[\frac{1}{3}t^3 + \frac{1}{5}t^5\right]_0^1 = \frac{48}{5}\pi$$