1. $x = t\sin t,\ y = t^2 + t \quad\Rightarrow\quad \dfrac{dy}{dt} = 2t + 1,\ \dfrac{dx}{dt} = t\cos t + \sin t$, and $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{2t+1}{t\cos t + \sin t}$.

2. $x = \dfrac{1}{t},\ y = \sqrt{t}\,e^{-t} \quad\Rightarrow\quad \dfrac{dy}{dt} = t^{1/2}(-e^{-t}) + e^{-t}\left(\frac{1}{2}t^{-1/2}\right) = \frac{1}{2}t^{-1/2}e^{-t}(-2t+1) = \dfrac{-2t+1}{2t^{1/2}e^t},\ \dfrac{dx}{dt} = -\dfrac{1}{t^2}$, and

$\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{-2t+1}{2t^{1/2}e^t}\left(-\dfrac{t^2}{1}\right) = \dfrac{(2t-1)t^{3/2}}{2e^t}$.

4. $x = t - t^{-1},\ y = 1 + t^2;\ t = 1.\quad \dfrac{dy}{dt} = 2t,\ \dfrac{dx}{dt} = 1 + t^{-2} = \dfrac{t^2+1}{t^2}$, and $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = 2t\left(\dfrac{t^2}{t^2+1}\right) = \dfrac{2t^3}{t^2+1}$.

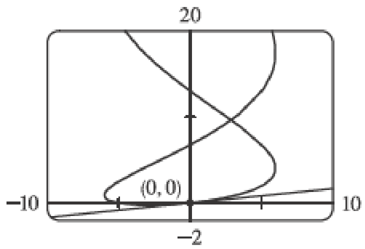
When $t = 1$, $(x, y) = (0, 2)$ and $dy/dx = \frac{2}{2} = 1$, so an equation of the tangent to the curve at the point corresponding to $t = 1$ is $y - 2 = 1(x - 0)$, or $y = x + 2$.

5. $x = e^{\sqrt{t}},\ y = t - \ln t^2;\ t = 1.\quad \dfrac{dy}{dt} = 1 - \dfrac{2t}{t^2} = 1 - \dfrac{2}{t},\ \dfrac{dx}{dt} = \dfrac{e^{\sqrt{t}}}{2\sqrt{t}}$, and $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{1 - 2/t}{e^{\sqrt{t}}/(2\sqrt{t})} \cdot \dfrac{2t}{2t} = \dfrac{2t-4}{\sqrt{t}e^{\sqrt{t}}}$.

When $t = 1$, $(x, y) = (e, 1)$ and $\dfrac{dy}{dx} = -\dfrac{2}{e}$, so an equation of the tangent line is $y - 1 = -\dfrac{2}{e}(x - e)$, or $y = -\dfrac{2}{e}x + 3$.

9. $x = 6\sin t,\ y = t^2 + t;\ (0, 0)$.

$\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{2t+1}{6\cos t}$. The point $(0, 0)$ corresponds to $t = 0$, so the slope of the tangent at that point is $\frac{1}{6}$. An equation of the tangent is therefore $y - 0 = \frac{1}{6}(x - 0)$, or $y = \frac{1}{6}x$.



11. $x = 4 + t^2,\quad y = t^2 + t^3 \quad\Rightarrow\quad \dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{2t + 3t^2}{2t} = 1 + \dfrac{3}{2}t \quad\Rightarrow$

$\dfrac{d^2y}{dx^2} = \dfrac{d}{dx}\left(\dfrac{dy}{dx}\right) = \dfrac{d(dy/dx)/dt}{dx/dt} = \dfrac{(d/dt)\left(1 + \frac{3}{2}t\right)}{2t} = \dfrac{3/2}{2t} = \dfrac{3}{4t}$.

The curve is CU when $\dfrac{d^2y}{dx^2} > 0$, that is, when $t > 0$.

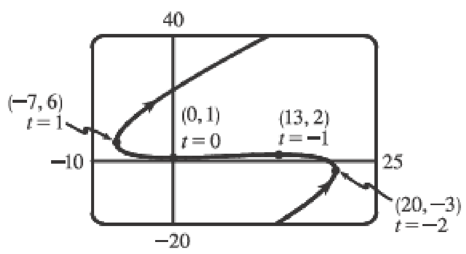
18. $x = 2t^3 + 3t^2 - 12t,\ y = 2t^3 + 3t^2 + 1$.

$\dfrac{dy}{dt} = 6t^2 + 6t = 6t(t+1)$, so $\dfrac{dy}{dt} = 0 \quad\Leftrightarrow$

$t = 0$ or $-1 \quad\Leftrightarrow\quad (x, y) = (0, 1)$ or $(13, 2)$.

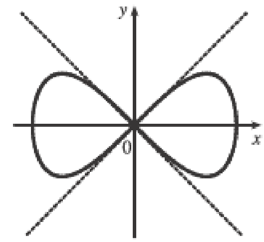
$\dfrac{dx}{dt} = 6t^2 + 6t - 12 = 6(t+2)(t-1)$, so $\dfrac{dx}{dt} = 0 \quad\Leftrightarrow$

$t = -2$ or $1 \quad\Leftrightarrow\quad (x, y) = (20, -3)$ or $(-7, 6)$.



The curve has horizontal tangents at $(0, 1)$ and $(13, 2)$, and vertical tangents at $(20, -3)$ and $(-7, 6)$.

25. $x = \cos t$, $y = \sin t \cos t$. $dx/dt = -\sin t$, $dy/dt = -\sin^2 t + \cos^2 t = \cos 2t$. $(x, y) = (0, 0) \Leftrightarrow \cos t = 0 \Leftrightarrow t$ is an odd multiple of $\frac{\pi}{2}$. When $t = \frac{\pi}{2}$, $dx/dt = -1$ and $dy/dt = -1$, so $dy/dx = 1$. When $t = \frac{3\pi}{2}$, $dx/dt = 1$ and $dy/dt = -1$. So $dy/dx = -1$. Thus, $y = x$ and $y = -x$ are both tangent to the curve at $(0, 0)$.



29. $x = 2t^3$, $y = 1 + 4t - t^2 \Rightarrow \dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{4 - 2t}{6t^2}$. Now solve $\dfrac{dy}{dx} = 1 \Leftrightarrow \dfrac{4 - 2t}{6t^2} = 1 \Leftrightarrow$ $6t^2 + 2t - 4 = 0 \Leftrightarrow 2(3t - 2)(t + 1) = 0 \Leftrightarrow t = \frac{2}{3}$ or $t = -1$. If $t = \frac{2}{3}$, the point is $\left(\frac{16}{27}, \frac{29}{9}\right)$, and if $t = -1$, the point is $(-2, -4)$.