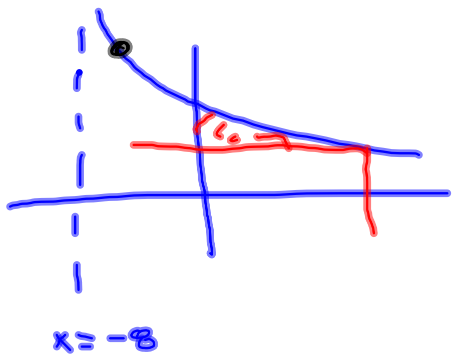
Test 3 I

If your Consumer Surplus Question was $p = \frac{33}{x+8}$, the question itself is bad and we need a re-do on that.



12.7

36 $\sum \frac{1}{(\ln(n))^{\ln(n)}}$ is a strange animal.

$$\sum_{n=2}^{\infty} \frac{1}{(\ln(n))^{\ln(n)}}$$

$$\ln(n)^{\ln(n)} = \left(e^{\ln(\ln(n))}\right)^{\ln(n)} \qquad (a^b)^c$$

$$= e^{(\ln(\ln(n)))(\ln(n))} \qquad a^{bc}$$

$$= e^{(\ln(n))(\ln(\ln(n)))} \qquad a^{cb}$$

$$= \left(e^{\ln(n)}\right)^{\ln(\ln(n))} \qquad (a^c)^b$$

$$= n^{\ln(\ln(n))}$$

So we're REALLY looking at:

$$\sum \frac{1}{n^{\ln(\ln(n))}}$$

I have a feeling that $\ln(\ln(n))$ is increasing.

"It is," saith the teacher.

$$\frac{1}{\quad}$$

Compare to a p-series?
If we can get $\ln(\ln(n)) > 1$, for all n, we're done!

$f(x) = \ln(\ln(x))$ $\quad \ln(x) \xrightarrow{x\to\infty} \infty$

$\ln(\ln(x)) \xrightarrow{x\to\infty} \infty$

So, at SOME point, $\ln(\ln(x)) > 2$ for all $x >$ something.

Same goes for $\ln(\ln(n))$. There's an $N \in \mathbb{N}$ such that $\ln(\ln(n)) > 2$ for all $n \geq N$.

Consider: $\displaystyle\sum_{n=N}^{\infty} \frac{1}{n^{\ln(\ln(n))}}$

Now, $\dfrac{1}{n^{\ln(\ln(n))}} < \dfrac{1}{n^2} \quad \forall n \geq N$

& $\displaystyle\sum_{n=N}^{\infty} \frac{1}{n^2}$ converges. So, $\displaystyle\sum_{n=N}^{\infty} \frac{1}{n^{\ln(\ln(n))}}$ converges.

$\therefore \displaystyle\sum_{n=1}^{\infty}$ mess converges.

② $\displaystyle\sum_{n=1}^{\infty} \frac{(2n+1)^n}{n^{2n}}$

Ratio Test you run into

$$\frac{(2(n+1)+1)^{n+1}}{(n+1)^{2(n+1)}} \cdot \frac{n^{2n}}{(2n+1)^n}$$ &

cancelling isn't very easy.

$$\sqrt[n]{\frac{(2n+1)^n}{(n^2)^n}} = \frac{2n+1}{n^2} = \frac{2+\frac{1}{n}}{n} \xrightarrow{n\to\infty} 0 < 1$$

$\therefore$ $\boxed{\text{Converges}}$ absolutely.

④ $\displaystyle\sum (-1)^n \frac{n}{n^2+2}$ Alternating Series Test

$\exists N \in \mathbb{N} \ni b_n \geq b_{n+1} \geq \cdots \ \forall n > N$ "Eventually decreasing."

(i) $b_1 \geq b_2 \geq \cdots$

(ii) $b_n \xrightarrow{n\to\infty} 0$

Need to argue that $\boxed{b_n = \frac{n}{n^2+2}}$ is decreasing.

$$f(x) = \frac{x}{x^2+2}$$

$$f'(x) = \frac{1(x^2+2) - x(2x)}{(x^2+2)^2} = \frac{-x^2+2}{(x^2+2)^2}$$ SOLVE < 0

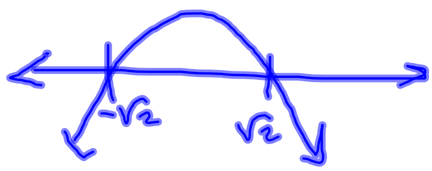
$\Longrightarrow$ $-x^2+2 < 0$

$-x^2+2=0$

$-x^2 = -2$

$x^2 = 2$

$x = \pm\sqrt{2}$



$-x^2+2$ sign pattern

$f'(x) < 0$ for $x > \sqrt{2}$

$\boxed{\text{So (i) } \checkmark}$

$\therefore$ $\frac{n}{n^2+2}$ is decreasing for $n > \sqrt{2}$, i.e., $n > 1$

(ii) $\displaystyle\frac{n}{n^2+2} = \frac{1}{n+\frac{2}{n}} \xrightarrow{n\to\infty} 0$

$\therefore$ The series converges.

(16) $\sum \frac{n^2+1}{n^3+1}$ Kinda like $\sum \frac{1}{n}$ if I could ignore the +1's.

This one, you could:

$\sum \frac{n^2+1}{n^3}$ $\quad \frac{n^2+1}{n^3} > \frac{n^2}{n^3} = \frac{1}{n}$ $\sum$

Direct comparison for #16 doesn't fly

$\sum \frac{1}{n}$ diverges. So $\sum \frac{n^2+1}{n^3}$ diverges.

So go LIMIT COMPARISON with $\sum \frac{1}{n}$

$$\frac{\frac{n^2+1}{n^3+1}}{\frac{1}{n}} = \frac{n^2+1}{n^3+1} \cdot \frac{n}{1} = \frac{n^3+n}{n^3+1}$$

$$= \frac{n^3\left(1+\frac{1}{n^2}\right)}{n^3\left(1+\frac{1}{n^3}\right)} = \frac{1+\frac{1}{n^2}}{1+\frac{1}{n^3}} \xrightarrow{n\to\infty} \frac{1}{1} = 1 \in \mathbb{R}$$

$\therefore$ They BOTH Diverge.

Limit Comparison not as elegant, but more powerful.