§12.6 #s 2–5, 8, 10, 12, 13, 18, 20, 23, 25, 27, 30, 35, 36

From last time:

3, 6, 12, 22, 26

#3 $\dfrac{(-10)^n}{n!}$

Ratio test.

$$\left|\frac{\frac{(-10)^{n+1}}{(n+1)!}}{\frac{(-10)^n}{n!}}\right| = \frac{10^{n+1}}{(n+1)!}\cdot\frac{n!}{10^n}$$

$$= \frac{10}{n+1} \xrightarrow{n\to\infty} 0 < 1$$ Absolutely Convergent.

6 $\sum \dfrac{(-1)^n}{n^4}$ converges ABSOLUTELY

since $\sum \frac{1}{n^4}$ passes urinalysis. P-TEST

12 $\sum \dfrac{\sin(4n)}{4^n}$ Converges Absolutely

$$\left|\frac{\sin(4n)}{4^n}\right| \le \frac{1}{4^n}$$ & $\sum \frac{1}{4^n}$ converges

26 You can see that we have the new factors in the numerator increasing by 4 and in the denominator increasing by 3.

$\frac{2}{5} + \frac{2}{5}\cdot\frac{6}{8}$ $\qquad\qquad \frac{2+4n}{5+3n}$

$a_1 = \frac{2}{5}$

$a_2 = a_1\cdot\frac{6}{8} = a_1\cdot\frac{2+4}{5+3} = a_1\cdot\frac{2+4(2-1)}{2+3(2-1)}$

$a_3 = a_2\cdot\frac{10}{11} = a_2\cdot\frac{2+4(3-1)}{5+3(3-1)} \quad \cdots$

$a_n = a_{n-1}\cdot\frac{2+4(n-1)}{5+3(n-1)} = \frac{2+4n-4}{5+3n-3}\cdot a_{n-1}$

$= \frac{4n-2}{3n+2}\cdot a_{n-1}$

$a_{20} = a_{19}\cdot\frac{4(20)-2}{3(20)+2} = \frac{78}{62}\cdot a_{19}$

& it just gets worser and worser.

$a_n \not\to 0$ Might

$\boxed{\text{Diverges}}$

13 $\sum \frac{10^n}{(n+1)\, 4^{2n+1}}$ $\qquad \frac{10^n}{(n+1)4^{2n}\cdot 4^1} = \frac{10^n}{4(n+1)\cdot 4^{2n}}$

Ratio Test also very good here.

$$= \frac{10^n}{4(n+1)\, 16^n}$$

$$= \frac{1}{4(n+1)} \cdot \left(\frac{10}{16}\right)^n$$

$$\leq \left(\frac{10}{16}\right)^n$$ compares favorably with convergent $\sum_{n=1}^{\infty} \left(\frac{5}{8}\right)^n$

21 $\sum_{n=1}^{\infty} \left(\frac{n^2+1}{2n^2+1}\right)^n$

$$\sqrt[n]{|a_n|} = \sqrt[n]{\left(\frac{n^2+1}{2n^2+1}\right)^n} = \frac{n^2+1}{2n^2+1} \xrightarrow{n\to\infty} \frac{1}{2}$$

So $\sum a_n$ is absolutely convergent.

25 $1 - \frac{1\cdot 3}{3!} + \frac{1\cdot 3\cdot 5}{5!} - \frac{1\cdot 3\cdot 5\cdot 7}{7!} + \cdots + (-1)^{n-1}\frac{1\cdot 3\cdot 5\cdots(2n-1)}{(2n-1)!}$

$n=1 \quad n=2 \quad n=3 \quad n=4$

Ratio test works.
Alternating Series Test works

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{1\cdot 3\cdot 5\cdots(2n-1)(2(n+1)-1)}{(2(n+1)-1)(2n)(2n-1)!}\cdot\frac{(2n-1)!}{1\cdot 3\cdot 5\cdots(2n-1)}$$

$$(2(n+1)-1)! = (2(n+1)-1)(2n)(2n-1)!$$

$$= \frac{1}{2n} \xrightarrow{n\to\infty} 0.$$

12.7 Stragety:

1. p-series
2. geometric series
3. Anything similar to 1 or 2 that you can compare or limit-compare.
4. $\lim_{n\to\infty} a_n \neq 0$ Diverge.
5. Alternating Series.
6. Factorials or other products $1\cdot3\cdot5\cdots(2n-1)$
   Ratio Test.
7. $a_n = (b_n)^n$, where you know something about b_n Root Test.
8. $a_n = f(n)$, where $\int_1^\infty f(x)\,dx$ is manageable.