12.4 # 12

$$\sum_{n=0}^{\infty} \frac{1+\sin n}{10^n} = \sum \left(\frac{1}{10^n} + \frac{\sin n}{10^n} \right)$$

If $\sum a_n$ & $\sum b_n$ converge, then so does $\sum (a_n + b_n)$

And ① $\sum \frac{1}{10^n}$ converges by Geometric Series, $r = \frac{1}{10}$.

② $\sum \frac{\sin n}{10^n}$ converges.

$$a_n = \frac{\sin n}{10^n} \leq \frac{|\sin n|}{10^n} \leq \frac{1}{10^n} = b_n$$

Trust me, it converges. Use Limit Comparison Test with $\frac{1}{10^n}$

By ① & ②, we have $\sum \frac{1+\sin n}{10^n}$ converge

$$|R_n| = \left| \sum_{k=n+1}^{\infty} (-1) b_k \right| \leq b_{n+1}$$

(i) $b_n \geq b_{n+1} \quad \forall n \in \mathbb{N}$
(ii) $b_n \xrightarrow{n \to \infty} 0$

$\Big\}$ Generally our working assumptions.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^6} \qquad \text{want } |R_n| \leq .00005$$

$$\text{want } \frac{1}{n^6} < .00005$$

$$\Longrightarrow \quad n^6 > \frac{1}{.00005} = \frac{1}{\frac{5}{100000}} = \frac{100000}{5} = 20000$$

$$\Longrightarrow \quad n > \sqrt[6]{20000} = 20000^{\frac{1}{6}}$$

$$\approx \underline{5.21000731 0}$$

$n = 6$ will do it.

55 outta 5 4's. $\quad 44 + \frac{44}{4}$

27–30 Approximate the sum of the series correct to four decimal places.

28. $\sum_{n=1}^{\infty} \frac{(-1)^n n}{8^n}$

See 12.5 Worksheet - By hand, you'd churn these out until you found a term that was less than .0001, and take the sum directly before.

32–34 For what values of p is each series convergent?

32. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^p}$

33. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n+p}$ $\mathbb{Z}^+ =$ nonnegative integers.

Convergent $\forall p \in \mathbb{R}$

34. $\sum_{n=2}^{\infty} (-1)^{n-1} \frac{(\ln n)^p}{n}$

I *think* we're just looking to see what it takes to make the sequence of terms to converge to zero.

34 Want to show $\frac{(\ln n)^p}{n} \xrightarrow{n\to\infty} 0$

Not really. Want condition(s) on p so that this holds.

$$\frac{(\ln n)^p}{n} \xrightarrow{n\to\infty}$$

Case 1: $p=0 \Rightarrow \frac{1}{n} \xrightarrow{n\to\infty} 0$ is clear.

(Alternating harmonic series ✓)

Case 2: $p>0$

$$\lim_{n\to\infty} \frac{(\ln n)^p}{n} = \frac{\infty}{\infty}$$

$$\lim_{n\to\infty} \frac{(\ln n)^p}{n} \overset{L'H}{=} \lim_{n\to\infty} \frac{p(\ln n)^{p-1}\left(\frac{1}{n}\right)}{1} = \lim_{n\to\infty} \frac{p(\ln n)^{p-1}}{n}$$

$$\overset{L'H}{=} \lim_{n\to\infty} \frac{p(p-1)(\ln(n))^{p-2}}{n} = \cdots$$

Continue until p is "exhausted"

E.g. $p=7$

$$\lim_{n\to\infty} \frac{(\ln n)^p}{n} = \cdots \lim_{n\to\infty} \frac{p(p-1)(p-2)\cdots(p-5)\ln n}{n}$$

$$\overset{L'H}{=} \lim_{n\to\infty} \frac{p(p-1)(p-2)\cdots(p-5)}{n} = 0$$

So $p>0$ ✓

Case 3: $p<0$? : $\frac{1}{(\ln n)^{-p} n}$

35. Show that the series $\Sigma (-1)^{n-1}b_n$, where $b_n = 1/n$ if n is odd and $b_n = 1/n^2$ if n is even, is divergent. Why does the Alternating Series Test not apply?