12.3 #20

$$\sum_{n=1}^{\infty} \frac{1}{n^2-4n+5}$$

Partial Fractions. Telescoping sum

Ask again after you've tried it.

#30 Find $p \ni \sum \frac{\ln n}{n^p}$ converges.

$$\frac{\ln 1}{1} + \frac{\ln 2}{2^p} + \frac{\ln 3}{3^p} + \frac{\ln 4}{4^p} + \cdots$$

when will $\int_1^{\infty} \frac{\ln x}{x^p}\,dx$ converge?

Integrate by parts.

$u = \ln x \rightarrow du = \frac{1}{x}dx$

$dv = x^{-p}dx \rightarrow v = \frac{x^{1-p}}{1-p}$

$$uv - \int v\,du = \frac{x^{1-p}\ln x}{1-p} - \int \frac{x^{1-p}}{1-p} \cdot \frac{1}{x}\,dx$$

$$= \lim_{t\to\infty} \left\{ \frac{1}{1-p} x^{1-p} \ln x \Big]_1^t - \int_1^t \frac{1}{1-p} \cdot \frac{1}{x^p}\,dx \right\}$$

certainly need $p > 1$

Let's look @

$$\lim_{t\to\infty} \frac{1}{1-p} t^{1-p} \ln t = \frac{1}{1-p} \lim_{t\to\infty} \left(\frac{\ln t}{t^{p-1}} \right) \overset{L'H}{=}$$

$$\frac{1}{1-p} \lim_{t\to\infty} \frac{\frac{1}{t}}{(p-1)t^{p-2}} = -\frac{1}{(p-1)^2} \lim_{t\to\infty} \left\{ \frac{1}{t^{p-1}} \right\}$$

Converges for $p > 1$ ✓

$$a_n = n\sin\left(\frac{1}{n}\right) = \frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}} \xrightarrow{n\to\infty} \frac{0}{0}$$

$\infty \cdot 0$

$$\text{"}=\text{"}\ \frac{0}{\frac{1}{\infty}}\ \text{"}=\text{"}\ \frac{0}{0}$$

L'Hôpital handles this

$$\overset{L'H}{=} \lim_{n\to\infty} \frac{-\frac{1}{n^2}\cos\left(\frac{1}{n}\right)}{-\frac{1}{n^2}} = \lim_{n\to\infty} \cos\left(\frac{1}{n}\right) = 1$$

NOTE 1 Although the condition $a_n \leqslant b_n$ or $a_n \geqslant b_n$ in the Comparison Test is given for all n, we need verify only that it holds for $n \geqslant N$, where N is some fixed integer, because the convergence of a series is not affected by a finite number of terms. This is illustrated in the next example.

$\ln\left(\frac{n}{2}\right)$

EXAMPLE 2 Test the series $\sum_{n=1}^{\infty} \frac{\ln n}{n}$ for convergence or divergence.

Text uses the term "ultimately." Other terms for this include "eventually" or "for all but finitely many."

Want to say it diverges. But it takes a while to compare directly to $\sum \frac{1}{n}$, but if $n \geq 3$, we have $\ln(n) \geq 1$, so we can say $a_n \geq b_n = \frac{1}{n}$ if $\sum b_n = \sum \frac{1}{n}$ and $n \geq 3$.

So by Comparison Test $\sum \frac{\ln(n)}{n}$ diverges, because $\sum \frac{1}{n}$ diverges.

$\lim_{n\to\infty} \frac{\ln n}{n} \xrightarrow{n\to\infty} 0$

3–32 Determine whether the series converges or diverges.

5. $\sum_{n=1}^{\infty} \frac{n+1}{n\sqrt{n}} = \sum_{n\geq 1}^{\infty}\left(\frac{n}{n^{3/2}} + \frac{1}{n^{3/2}}\right) = \sum_{n\geq 1}^{\infty}\left(\frac{1}{\sqrt{n}} + \frac{1}{n^{3/2}}\right)$

$\sum a_n = \sum \frac{1}{\sqrt{n}}$ diverges.

$\sum b_n = \sum \frac{1}{n^{3/2}}$ converges

So $\sum (a_n + b_n)$ diverges.

$\int$ 12.2 #s 69, 70.

6. $\sum_{n=1}^{\infty} \frac{n-1}{n^2\sqrt{n}}$

(69) $\sum a_n$ converges & $\sum b_n$ diverges. ↳ Suppose true

Then $\sum(a_n+b_n)$ diverges. ↳ Assume false.

Proof (By contradiction) Conclude: ABSURDITY.

Suppose $\sum(a_n+b_n)$ is convergent.

By T8iii, we have

$$\sum(a_n+b_n) - \sum a_n = \sum(a_n+b_n-a_n)$$

$= \sum b_n$ converges. But by hypothesis, $\sum b_n$ diverges! $\therefore$ $\sum(a_n+b_n)$ is DIVERGENT

RAA arguments.

Reductio ad Absurdum.

$$\sum 0 = 0$$

$$\sum(1-1) = 0$$

But you can't say $\sum 1 + \sum(-1) = 0$

Because $\infty - \infty$ is indeterminate, at the least!

12. $\sum_{n=0}^{\infty} \frac{1 + \sin n}{10^n}$

ESTIMATING SUMS

The idea is that you can use the "nicer" series, that you used to compare to your "ugly" series to get a handle on the error in the first n terms of your ugly series.

If you used the comparison test on $s = \sum_{k=1}^{\infty} a_k$,

comparing it to $t = \sum_{k=1}^{\infty} b_k$, which converges, *and has all positive terms* (or eventually does...), then from the fact that $a_n \leq b_n$, we find that $R_n \leq T_n$, where

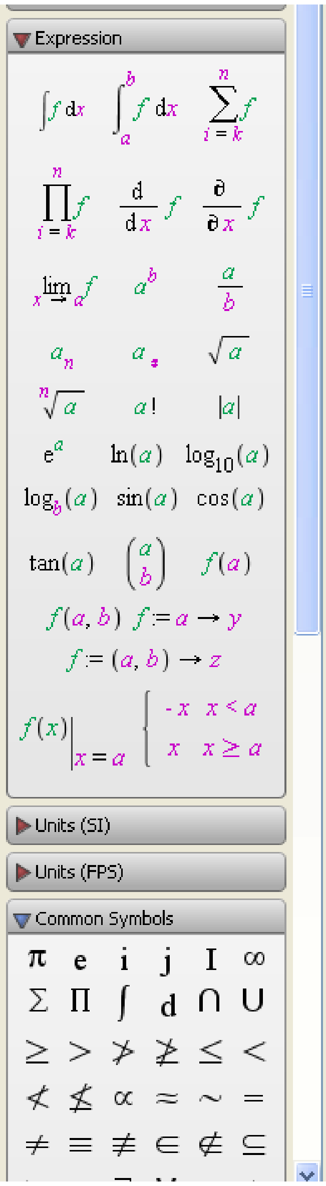
$$R_n = s - s_n = \sum_{k=n+1}^{\infty} a_k \text{ and } T_n = t - t_n = \sum_{k=n+1}^{\infty} b_k .$$

Since you usually pick a series that's EASIER to compare with, usually the integration estimates we make are EASIER to make for the T_n.

33–36 Use the sum of the first 10 terms to approximate the sum of the series. Estimate the error.

36. $\sum_{n=1}^{\infty} \frac{n}{(n+1)3^n}$

	A	B	C
1	k	a_k	s_k
2	1	0.166666667	0.166666667
3	2	0.074074074	0.240740741
4	3	0.027777778	0.268518519
5	4	0.009876543	0.278395062
6	5	0.003429355	0.281824417
7	6	0.001175779	0.283000196
8	7	0.000400091	0.283400287
9	8	0.000135481	0.283535768
10	9	4.57247E-05	0.283581493
11	10	1.53955E-05	0.283596888
12	11	5.17461E-06	0.283602063
13	12	1.73693E-06	0.2836038
14	13	5.82424E-07	0.283604382
15	14	1.95137E-07	0.283604577
16	15	6.5336E-08	0.283604643
17	16	2.18641E-08	0.283604665
18	17	7.31333E-09	0.283604672
19	18	2.44532E-09	0.283604674
20	19	8.17372E-10	0.283604675
21	20	2.7314E-10	0.283604676
22	21	9.12537E-11	0.283604676
23	22	3.04809E-11	0.283604676
24	23	1.01795E-11	0.283604676
25	24	3.39908E-12	0.283604676
26	25	1.13484E-12	0.283604676
27	26	3.78841E-13	0.283604676
28	27	1.26454E-13	0.283604676
29	28	4.22051E-14	0.283604676



$$\int_{10}^{\infty} \frac{x}{(x+1)\cdot 3^x}\,dx$$

$$-\frac{1}{59049}\,\frac{177147\,\mathrm{Ei}(1, 11\ln(3))\ln(3) - 1}{\ln(3)}$$

evalf(%)

$$0.00001411402009$$

$$\int_{10}^{\infty} \frac{1}{3^x}\,dx$$

$$\frac{1}{59049\ln(3)}$$

evalf(%)

$$0.00001541498123$$

This one is more for *my* fun. At worst, it'll be a bonus question on the next test. Speaking of which, how do we want to handle this? We're getting ahead of schedule, here. Might want to stop and do some reviewing before we plunge too deeply into the later material in Chapter 12.

One issue students have with Chapter 12 is that the different tests, especially the Ratio Test -versus- Limit Comparison Test, get confused in their minds... I'd like to try to structure things to avoid that, as best I might.

39. Prove that if $a_n \geq 0$ and $\Sigma\, a_n$ converges, then $\Sigma\, a_n^2$ also converges.