§12.2 Series — Sums of sequences.

$a_1, a_2, \ldots, a_n, \ldots$ is a sequence

$a_1 + a_2 + a_3 + \cdots + a_n + \cdots$ is the corresponding series.

Notation $\sum_{n=1}^{\infty} a_n = \sum a_n$ is OK, as long as context is clear.

Given $S = \sum_{n=1}^{\infty} a_n$,

$S_1 = a_1$

$S_2 = a_1 + a_2$

$S_3 = a_1 + a_2 + a_3$

$\vdots$

$S_n = a_1 + a_2 + a_3 + \cdots + a_n$ is the $n^{\underline{th}}$ partial sum. We say that $\sum_{n=1}^{\infty} a_n$ converges if the sequence of $n^{\underline{th}}$ partial sums $\{S_n\}$ converges, and we write $\sum_{n=1}^{\infty} a_n = L$ if $S_n \xrightarrow{n \to \infty} L$.

Geometric Series

$$a + ar + ar^2 + \cdots + ar^{n-1} + \cdots$$

$n=1 \quad n=2$

$$= \sum_{n=1}^{\infty} ar^{n-1}$$ we find the sum.

$$S_n = a + ar + ar^2 + \cdots + ar^{n-1}$$

$$- \quad rS_n = \quad ar + ar^2 + \cdots + ar^{n-1} + ar^n$$

$$S_n - rS_n = a - ar^n$$

$$S_n[1-r] = a - ar^n$$

$$S_n = \frac{a(1-r^n)}{[1-r]}$$ is a nice closed-form expression for S_n.

$r^n \to 0$ as $n \to \infty$

Converges if $r \neq 1$

$r > 1$ Bad

$r \leq -1$ Bad

$|r| < 1$

S 12.2 I #s 1, 2, 3, 7–12, 16, 19, 20, 22, 25, 29–31

12.2 II #s 35, 36, 41, 42, 47, 48, 52, 65,

69, 70

See TB discussion.

$$0 = 1 - 1 + 1 - 1 + 1 - 1 + \cdots$$
$$= 1 + (-1 + 1) + (-1 + 1) + (-1 + 1) + \cdots$$
$$= 1$$