$$\frac{1}{1}, \frac{1}{3}, \frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{1}{5}, \frac{1}{4}, \frac{1}{6}, \ldots$$

$k=1$ $n=1$ $\frac{1}{1}$

$k=2$ $n=3$ $\frac{1}{2}$ $2 = \frac{3+1}{2}$

$k=3$ $n=5$ $\frac{1}{3}$ $3 = \frac{5+1}{2}$

$k=4$ $n=7$ $\frac{1}{4}$ $4 = \frac{7+1}{2}$

$$\boxed{\begin{array}{c} n = 2k-1 \\ \dfrac{1}{\frac{n+1}{2}} \end{array}} \quad \text{for } k = 1, 2, \ldots$$

$$= \frac{2}{n+1}$$

$n=2$ $\frac{1}{3}$

$n=4$ $\frac{1}{4}$

$n=6$ $\frac{1}{5}$

$n=8$ $\frac{1}{6}$

$n = 2k$ for $k \in \mathbb{N}$

$$a_n = \begin{cases} \dfrac{2}{n+1} & \text{if } n \text{ is odd} \\ \dfrac{2}{n+4} & \text{if } n \text{ is even} \end{cases}$$

$2k+1$, $2k-1$

$2k$

$$a_n = \frac{1}{\frac{n+4}{2}} = \frac{2}{n+4}$$, Joel.

$\boxed{\text{T12.1.9}}$ r^n converges if $-1 < r \le 1$ & diverges W

$$\left(\tfrac{1}{3}\right)^n \xrightarrow{n\to\infty} 0 \qquad \left(\tfrac{4}{3}\right)^n \not\longrightarrow$$

$$1^n \xrightarrow{n\to\infty} 1$$

$\boxed{\text{D 10}}$ $\left.\begin{array}{l}\{a_n\} \text{ is increasing} \\ \{a_n\} \text{ is decreasing}\end{array}\right\}$ Montonic

$\boxed{\text{Recall}}$ $\{a_n\}$ is Bdd means

$a_n \le M$ for some $M \in \mathbb{R}$ and for all $n \in \mathbb{N}$.

Bdd Above

Bdd Below

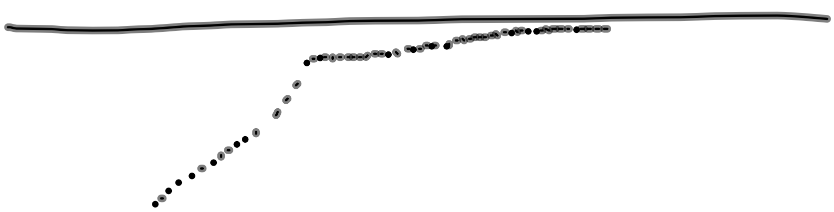
$a_n \ge M$ for some $M \in \mathbb{R}$ and $\forall n \in \mathbb{N}$

Bdd above & below : BOUNDED

$\exists M \ni |a_n| \le M \quad \forall n \in \mathbb{N}.$

$\boxed{\text{T 12.1.12}}$ Bdd & Monotone $\Longrightarrow$ Convergent.

Monotone Convergence Theorem.



#44 Converges to zero

$$a_n = \begin{cases} \dfrac{2}{n+1} & n \text{ is odd} \\ \dfrac{2}{n+4} & n \text{ is even} \end{cases}$$

$$\left.\begin{array}{l} \dfrac{2}{n+1} \xrightarrow{n \to \infty} 0 \\ \dfrac{2}{n+4} \xrightarrow{n \to \infty} 0 \end{array}\right\} \therefore \; a_n \xrightarrow{n \to \infty} 0.$$

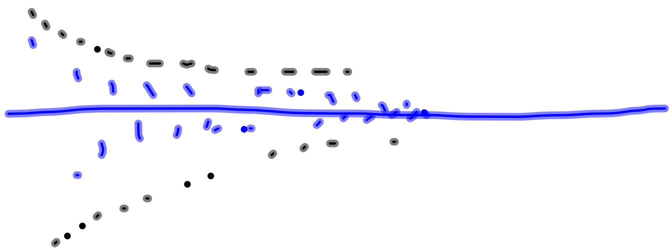
40 $a_n = \dfrac{\sin(2n)}{1+\sqrt{n}}$

$$-1 \le \sin(2n) \le 1$$

$$\frac{-1}{1+\sqrt{n}} \le \frac{\sin(2n)}{1+\sqrt{n}} \le \frac{1}{1+\sqrt{n}}$$

$$\frac{-1}{1+\sqrt{n}} \xrightarrow{n \to \infty} 0 \qquad \frac{1}{1+\sqrt{n}} \xrightarrow{n \to \infty} 0$$

By squeeze Theorem, it converges to zero.



64 $a_n = ne^{-n}$ Increasing? Decreasing? Bdd?

$\underline{f(x) = xe^{-x}}$

xe^{-x}

$\frac{e^x}{x} \xrightarrow{x \to \infty} \infty$

$\frac{x}{e^x} \xrightarrow{x \to \infty} 0$

Bdd: ?

Decreasing is shown, then bdd below will suffice in showing convergence.

$\boxed{\text{A trick}}$

$f(x) = xe^{-x}$

$\Longrightarrow f'(x) = e^{-x} - xe^{-x} = e^{-x}(1-x)$

$+ \quad | \quad - \quad f'$

1

This suggest a_n is monotone decreasing.

$\boxed{a_1 > a_2}$

Since $ne^{-n} > 0 \ \forall n \geq 1$

$\boxed{\text{We have convergence}}$