## §12.1 Sequences

$\boxed{D}$ A sequence is a list of #s written in a particular order.

$$a_1, a_2, a_3, \ldots$$

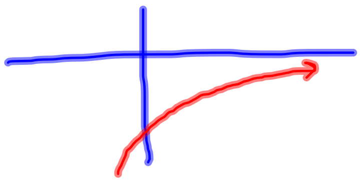
$\boxed{\text{Alternate}}$ A sequence is a function from $\mathbb{N}$ into $\mathbb{R}$, i.e., $f: \mathbb{N} \longrightarrow \mathbb{R}$

$$a_1 = f(1),\ a_2 = f(2), \ldots$$

This alternate lets us use what we know about functions from $\mathbb{R}$ into $\mathbb{R}$ to say things about sequences. $f: \mathbb{R} \longrightarrow \mathbb{R}$

$\boxed{\text{Notation}}$ $\{a_1, a_2, a_3, \ldots\} = \{a_n\} = \{a_n\}_{n=1}^{\infty}$

$\boxed{\text{Examples}}$ $\left\{\dfrac{\ln(n)}{n}\right\}$



Application of "Alternate"

Fact: $\dfrac{\ln(n)}{n}$ converges to $0$.

$$\lim_{x\to\infty} \frac{\ln x}{x} \overset{\text{L'HOP}}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = 0$$

$$\{n\} = \{1, 2, 3, \ldots\}$$

$$\left\{(-1)^n \cdot \frac{1}{n}\right\} = \left\{-1, \frac{1}{2}, -\frac{1}{3}, \frac{1}{4}, \ldots\right\}$$

$$\left\{(-1)^{n+1} \cdot \frac{1}{n}\right\} = \left\{1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \ldots\right\} = \left\{(-1)^{n-1} \cdot \frac{1}{n}\right\}$$

Fun Puzzles  Find $a_n$ in general:

#10 $\{1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots\}$

$n=1$  $1 = \frac{1}{1}$  $\frac{1}{n}$?  Guess $\frac{3}{3^n}$?  $\left.\begin{matrix}\frac{3}{3^1} \\ \frac{3}{3^2}\end{matrix}\right\}$ David's

$n=2$  $\frac{1}{2}$ Nope  $\frac{1}{3} = \frac{1}{3^1} = \frac{1}{3^{2-1}}$  $\frac{1}{3^{n-1}}$

$n=3$  $\frac{1}{9} = \frac{3}{3^3} = \frac{1}{3^{3-1}} = \frac{1}{3^{n-1}}$

$n=4$  $\frac{1}{27} = \frac{1}{3^3} = \frac{1}{3^{4-1}} = \frac{1}{3^{n-1}}$

$$\boxed{a_n = \frac{1}{3^{n-1}}}$$

#13  $n=1$, $n=2$, $n=3$, $n=4$

$1, -\frac{2}{3}, \frac{4}{9}, -\frac{8}{27},$

$(-1)^{n+1}$

$n=1$: $\frac{2^{n-1}}{3^{n-1}} \cdot (-1)^{n+1} = \frac{2^{n-1}}{3^{n-1}} \cdot (-1)^{n-1} = \left(-\frac{2}{3}\right)^{n-1}$

$\boxed{D}$ $\{a_n\}$ has limit $L$ and we write

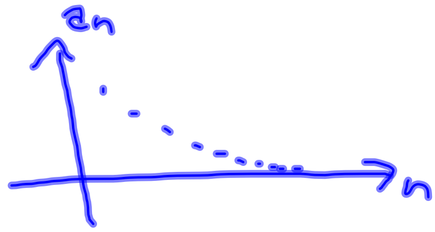
$\lim_{n\to\infty} a_n = L$ OR $a_n \to L$ as $n \to \infty$

OR $a_n \xrightarrow{n\to\infty} L$

Means
if we can make $a_n$ as close to $L$ as we like by taking $n$ sufficiently large.
(And every $a_n$ after that will also be within the desired tolerance.)

If $a_n \xrightarrow{n\to\infty} L$, $\{a_n\}$ converges.

W diverges.

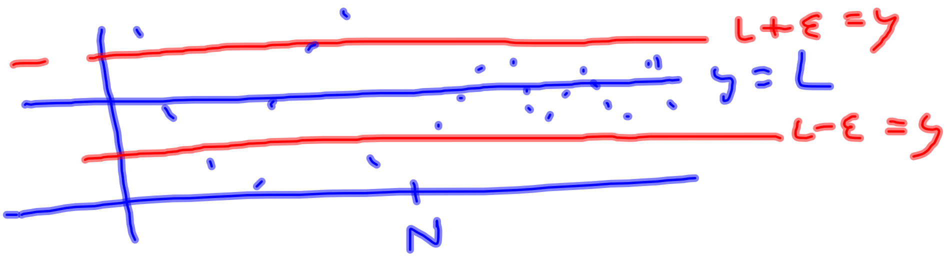



$\boxed{E}$ Harmonic Sequence

$\{\frac{1}{n}\}$ . $\frac{1}{n} \xrightarrow{n\to\infty} 0$

$\boxed{D2}$ $\{a_n\}$ has limit $L$, i.e., $a_n \xrightarrow{n\to\infty} L$
iff

$\forall \varepsilon > 0, \exists N \in \mathbb{N} \ni$ if
$n > N$ then $|a_n - L| < \varepsilon$.




$(a_n \xrightarrow{n\to\infty} L)$ iff $\Longleftrightarrow$ $(\forall \varepsilon > 0\ \exists N \in \mathbb{N} \ni n > N \Rightarrow |a_n - L| < \varepsilon)$

#15 $1^{st}$ 6 terms of $\{a_n\} = \{\frac{n}{2n+1}\}$

Does it converge?

i.e., does $f(x) = \frac{x}{2x+1}$ have a HORIZONTAL ASYMPTOTE.

yes, $y = \frac{1}{2}$

$\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{5}{11}, \frac{6}{13}$

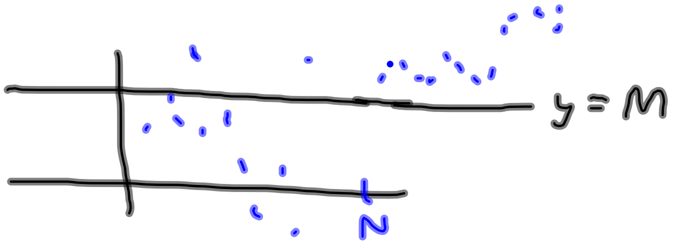
Yes $a_n \xrightarrow{n\to\infty} \frac{1}{2}$

$$\frac{n}{2n+1} = \frac{n(1)}{n(2+\frac{1}{n})} = \frac{1}{2+\frac{1}{n}} \xrightarrow{n\to\infty} \frac{1}{2}$$

$\boxed{T3}$ $f(x) \xrightarrow{x\to\infty} L$ & $a_n = f(n)$

Then $a_n \xrightarrow{n\to\infty} L$

$\boxed{D5}$ $a_n \xrightarrow{n\to\infty} \infty$ means if you give me $M > 0$, I can find $N \in \mathbb{N}$ such that $a_n > M$ for every $n > N$.



Limit Laws $c \in \mathbb{R}$

$\lim (a_n + b_n)$, $\lim (a_n - b_n)$, $\lim (c\, a_n)$,

$\lim (a_n b_n)$, $\lim \left(\frac{a_n}{b_n}\right)$, $\lim (a_n^p)$

$p > 0$, $a_n > 0$

$\lim (a_n^p) = \left(\lim (a_n)\right)^p$

$\boxed{\text{J7}}$ If $a_n \xrightarrow{n \to \infty} L$ & $f$ is cnt^s, then

$$\lim_{n\to\infty} f(a_n) = f(L)$$
$$= f\left(\lim_{n\to\infty} a_n\right)$$

1, 2, 3, 5, 7 – 45 ODDS, 47, 49, 51, 53, 56? 57,
61, 63, 67!

Honest Effort.

$5 \bmod 2 = 1$

$6 \bmod 2 = 0$

$n=1 \quad \frac{1}{1}$ , $\qquad n=2 \quad \frac{1}{3}$

$n=3 \quad \frac{1}{2} \qquad n=4 \quad \frac{1}{4}$

$n=5 \quad \frac{1}{3} \qquad n=6 \quad \frac{1}{5}$

$n=8 \quad \frac{1}{6}$

$a_n = \frac{1}{k}$ if $n = 2k-1$ $\qquad n=10 \quad \frac{1}{7}$

$\frac{1}{2k}$

$10 =$