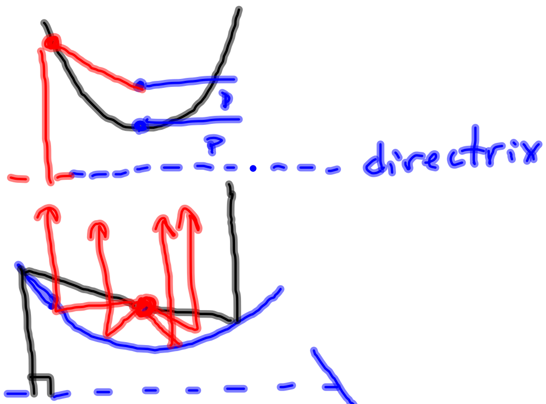
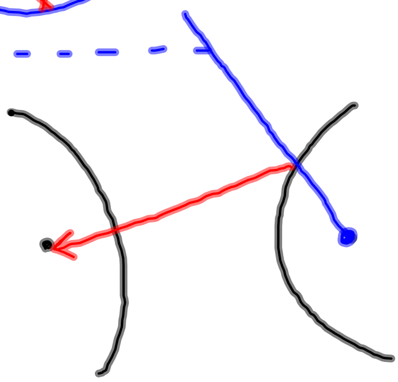


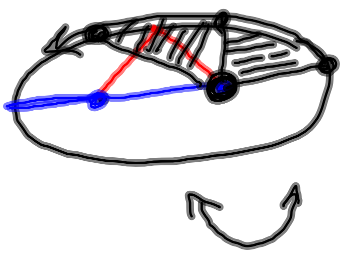
$4py = x^2$

Distance from vertex to focus.

$4px = y^2$



Hyperbola



Kepler's Laws Pg 702

Equal Areas

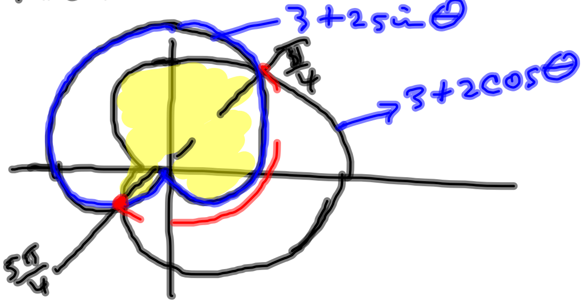
Equal times

11.6

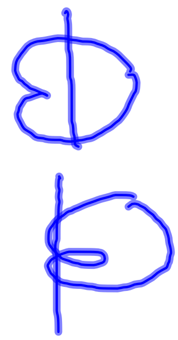
32. $r = 3 + 2\cos\theta$, $r = 3 + 2\sin\theta$

$r = 3 + 2\cos\theta$, $r = 3 + 2\sin\theta$

Area contained inside of both



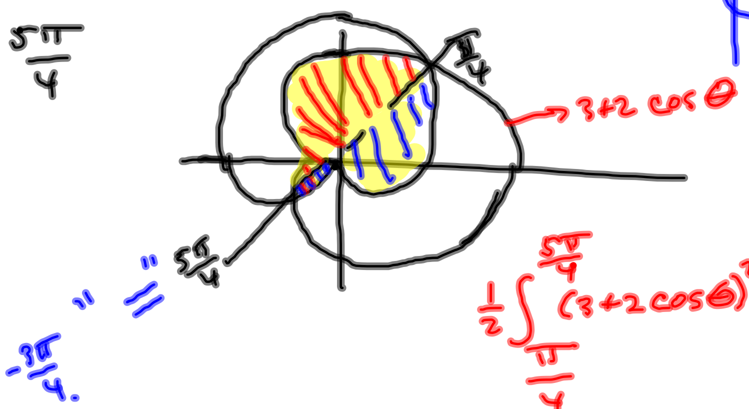
$2 + \underline{3\cos\theta}$



$2\sin\theta = 2\cos\theta$

$\tan\theta = 1$

$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$



$$\frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (3+2\cos\theta)^2\, d\theta$$

$$\cancel{\frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (3+2\sin\theta)^2\, d\theta}$$

$$\frac{1}{2}\int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}} (3+2\sin\theta)^2\, d\theta = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (3+2\cos\theta)^2\, d\theta \ !?$$

Symmetry.

36. Find the area between a large loop and the enclosed small loop of the curve $r = 1 + 2\cos 3\theta$.

37–42 Find all points of intersection of the given curves.

41. $r = \sin\theta, \quad r = \sin 2\theta$

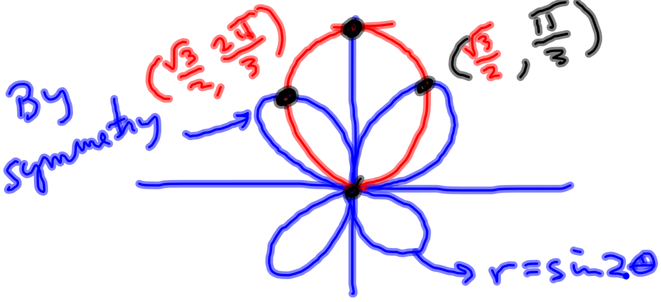
$\sin\theta = \sin(2\theta) \overset{*}{=} 2\sin\theta\cos\theta$ *

$2\sin\theta\cos\theta - \sin\theta = 0$

$\sin\theta(2\cos\theta - 1) = 0$

$\sin\theta = 0$ OR $\cos\theta = \frac{1}{2}$

$\theta = 0, \pi, 2\pi$ $\qquad \theta = \frac{\pi}{3}, \boxed{\frac{5\pi}{3}}$



$r = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$r = \sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$(0,0), \left(\frac{\pi}{3}, \frac{\sqrt{3}}{2}\right)$

$\left(\frac{2\pi}{3}, \frac{\sqrt{3}}{2}\right)$

ARC LENGTH

$(r, \theta) \to (x, y)$

$x = r\cos\theta = f(\theta)\cos\theta \qquad y = r\sin\theta = f(\theta)\sin\theta$

$\frac{dx}{d\theta} = \frac{dr}{d\theta}\cos\theta - r\sin\theta \qquad \frac{dy}{d\theta} = \frac{dr}{d\theta}\sin\theta + r\cos\theta$

$(a-b)^2 = a^2 - 2ab + b^2$

By Theorem 11.2.6
Arc Length is given by L:

$$L = \int_a^b \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\, d\theta$$

$$\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = \left(\frac{dr}{d\theta}\right)^2\cos^2\theta - 2r\frac{dr}{d\theta}\sin\theta\cos\theta + r^2\sin^2\theta$$

$$+ \left(\frac{dr}{d\theta}\right)^2\sin^2\theta + 2r\frac{dr}{d\theta}\sin\theta\cos\theta + r^2\cos^2\theta$$

$$= \left(\frac{dr}{d\theta}\right)^2\left[\cos^2\theta + \sin^2\theta\right] + r^2\left[\sin^2\theta + \cos^2\theta\right]$$

$$= \left(\frac{dr}{d\theta}\right)^2 + r^2$$

$$\boxed{L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta}$$

45–48 Find the exact length of the polar curve.

46. $r = e^{2\theta}$, $0 \leqslant \theta \leqslant 2\pi$

$$\frac{dr}{d\theta} = 2e^{2\theta}$$

$$L = \int_0^{2\pi} \sqrt{(e^{2\theta})^2 + (2e^{2\theta})^2}\, d\theta$$

$$= \int_0^{2\pi} \sqrt{e^{4\theta} + 4e^{4\theta}}\, d\theta$$

$$= \int_0^{2\pi} \sqrt{5e^{4\theta}}\, d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} \sqrt{5}\, e^{2\theta}\, 2\, d\theta = \frac{\sqrt{5}}{2}\left[e^{2\theta}\right]_0^{2\pi}$$

$$= \frac{\sqrt{5}}{2}\left[e^{4\pi} - 1\right]$$

49–52 Use a calculator to find the length of the curve correct to four decimal places.

50. $r = 4 \sin 3\theta$