11.4 #s 1, 4, 7, 10, 13, 15, 20, 26, 31, 35, 39, 42, 45, 52

11.4 AREAS AND LENGTHS IN POLAR COORDINATES

$A = \pi r^2$
$= \frac{1}{2} r^2 \cdot 2\pi$
$\frac{1}{2} r^2 \cdot \pi$
$\frac{1}{2} r^2 \cdot \frac{\pi}{2}$

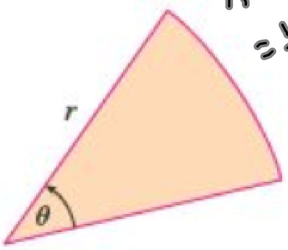


FIGURE 1

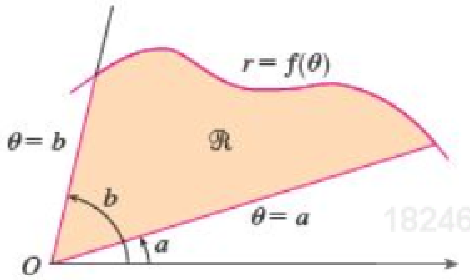


FIGURE 2

$$\underline{A = \frac{1}{2} r^2 \theta}$$

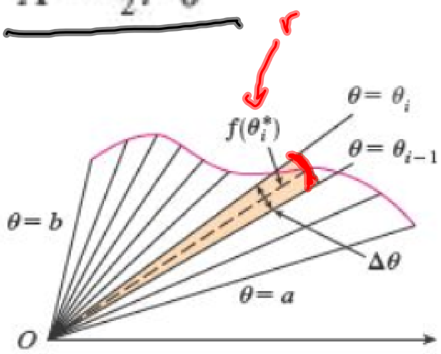


FIGURE 3

$$\lim_{n \to \infty} \sum_{i=1}^{n} \boxed{\frac{1}{2} [f(\theta_i^*)]^2 \, \Delta\theta} = \boxed{\int_a^b \frac{1}{2} [f(\theta)]^2 \, d\theta}$$

$$\boxed{A = \int_a^b \frac{1}{2} r^2 \, d\theta}$$

$$\boxed{A = \int_a^b \frac{1}{2} [f(\theta)]^2 \, d\theta}$$

EXAMPLE 2 Find the area of the region that lies inside the circle $r = 3\sin\theta$ and outside the cardioid $r = 1 + \sin\theta$.

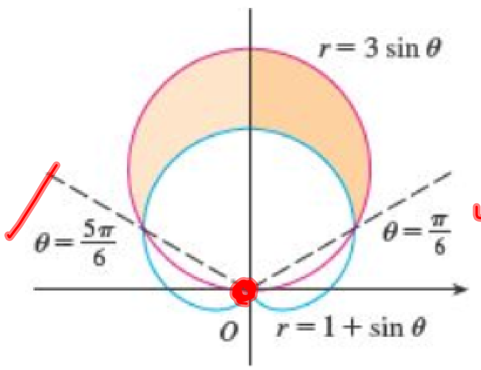
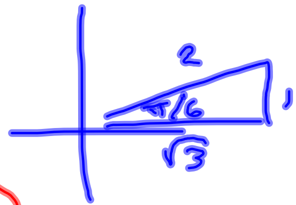
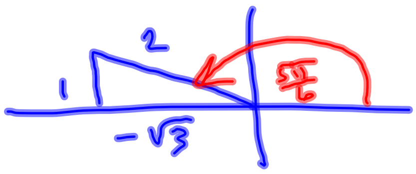


FIGURE 5

$$3\sin\theta = 1 + \sin\theta$$

$$\begin{cases} 2\sin\theta = 1 \\ \sin\theta = \frac{1}{2} \end{cases}$$





$$\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (3\sin\theta)^2\, d\theta - \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (1+\sin\theta)^2\, d\theta$$

$$= \frac{1}{2}\cdot 2 \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left[9\sin^2\theta - (1 + 2\sin\theta + \sin^2\theta)\right] d\theta$$

$$= 8\int_{\frac{\pi}{6}}^{\pi/2} \sin^2\theta\, d\theta - \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (1 + 2\sin\theta)\, d\theta \text{, etc.}$$

$r = 3|\sin\theta|$ is a figure 8!

The general idea:

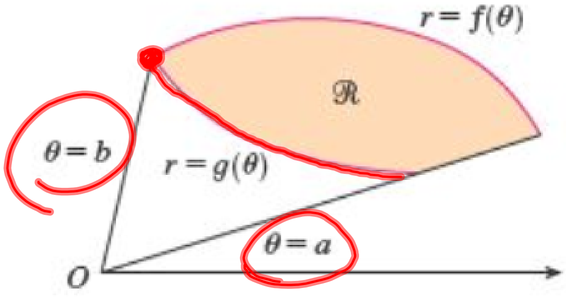


FIGURE 6

Area of $\mathcal{R} = A = \int_a^b \frac{1}{2}[f(\theta)]^2\,d\theta - \int_a^b \frac{1}{2}[g(\theta)]^2\,d\theta = \frac{1}{2}\int_a^b \left([f(\theta)]^2 - [g(\theta)]^2\right) d\theta$

CAUTION

Example 2

" the origin has no single representation in polar coordinates that satisfies both equations. "

Algebraic techniques won't always reveal a key point. One trick: Substitute - r for r since sometimes the point of intersection will have an angle that's, for instance, π radians (or some multiple thereof), and r is negative!

But the *best* way is to be clear on what the picture is and determine the relevant values for each function. Technology is a HUGE help in this!!! (Wish I'd had it when *I* took calculus. This stuff would've been a *whole* lot less mysterious!)

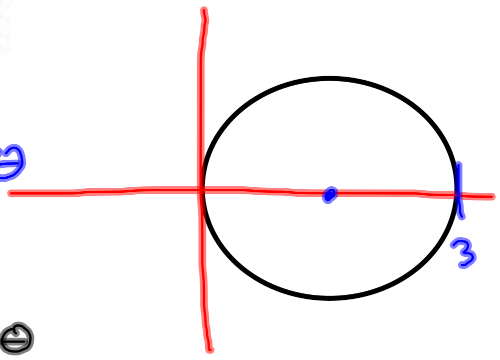
1–4 Find the area of the region that is bounded by the given curve and lies in the specified sector.

5–8 Find the area of the shaded region.

9–14 Sketch the curve and find the area that it encloses.

9. $r = 3\cos\theta$

$(1.5)^2\pi = \text{Area}$



$$\frac{1}{2}\int_0^{2\pi}(3\cos\theta)^2\,d\theta$$

$$= \frac{1}{2}\int_0^{\pi} 9\cos^2\theta\,d\theta$$

$$\frac{9}{2}\int_0^{\pi}\frac{1+\cos(2\theta)}{2}\,d\theta = \frac{9}{4}\int_0^{\pi} d\theta + \frac{9}{8}\int_0^{\pi}\cos(2\theta)\,2d\theta$$

$$= \frac{9}{4}\theta\Big]_0^{\pi} + \frac{9}{8}\Big[\sin(2\theta)\Big]_0^{\pi}$$

$$= \frac{9}{4}\pi = \left(\frac{3}{2}\right)^2\pi = (1.5)^2\pi$$

17–21 Find the area of the region enclosed by one loop of the curve.

18. $r = 4\sin 3\theta$

pole — r by $-r$

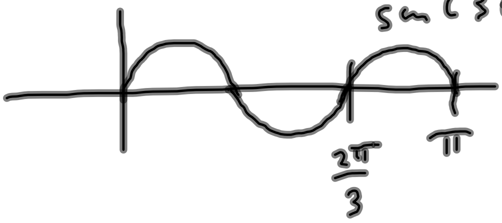
polar axis — θ by $-\theta$

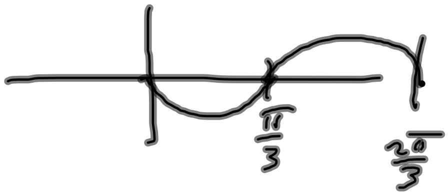
$\theta = \frac{\pi}{2}$ — θ by $\pi - \theta$

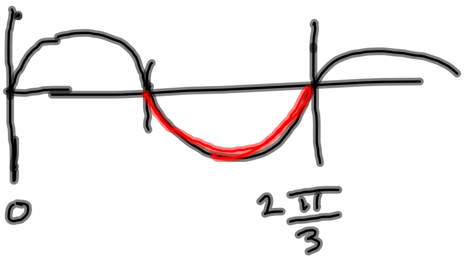
$4\sin(3(\pi-\theta))$

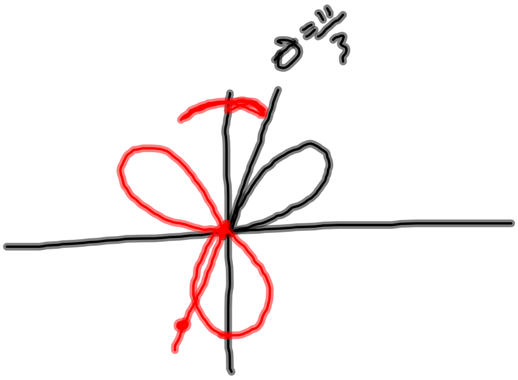
$= 4\sin(3\pi - 3\theta) = 4\sin(\pi - 3\theta) \overset{?}{=} 4\sin(3\theta)$?

$\sin(3\theta)$ — $\sin(-3\theta)$







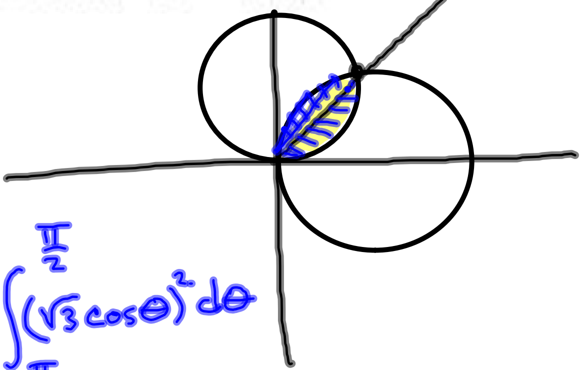


$$\frac{1}{2}\int_0^{\frac{\pi}{3}} (4\sin(3\theta))^2\, d\theta$$

29–34 Find the area of the region that lies inside both curves.

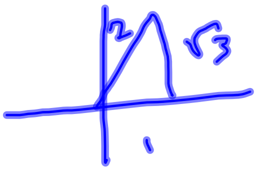
29. $r = \sqrt{3}\cos\theta, \quad r = \sin\theta$

$\theta = \frac{\pi}{3}$



$\sin\theta = \sqrt{3}\cos\theta$

$\tan\theta = \sqrt{3}$



$$\text{Area} = \frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (\sqrt{3}\cos\theta)^2\,d\theta + \frac{1}{2}\int_{0}^{\frac{\pi}{3}} (\sin\theta)^2\,d\theta$$