11

PARAMETRIC EQUATIONS AND POLAR COORDINATES

Section 11.1 #s 2, 3, 5, 6, 12, 13, 14, 17, 19, 20, 25, 26

2. use $t = 0, \pi/2, \pi, 3\pi/2, 2\pi$
3. use $t = -\pi, -\pi/2, 0, \pi/2, \pi$

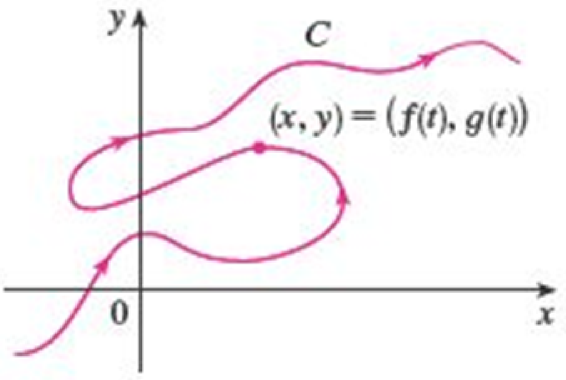


FIGURE 1

An example where you can "eliminate the parameter" t :

$$x = t^2 - 2t \qquad y = t + 1$$

Another Example where you can eliminate the parameter: The unit circle!

$$x = \cos t \qquad y = \sin t \qquad 0 \leqslant t \leqslant 2\pi$$

$$(x, y) = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$(x, y, z) = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix}$$

$$x = t^2 - 2t \qquad y = t + 1$$

$\Longrightarrow y - 1 = t$

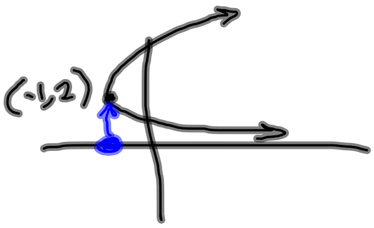
$x = (y-1)^2 - 2(y-1)$

$= y^2 - 2y + 1 - 2y + 2$

$= y^2 - 4y + 3$

$= y^2 - 4y + 2^2 - 4 + 3$

$= (y-2)^2 - 1$



$x = y^2$

$x = (y-2)^2$

$x = (y-2)^2 - 1$

weird, shifting $x = f(y)$ around.

$y = f(x)$

$\underline{y = f(x-2) - 1}$

$$x = \cos t \qquad y = \sin t \qquad 0 \leqslant t \leqslant 2\pi$$

$x^2 = \cos^2 t$, $y^2 = \sin^2 t$

$x^2 + y^2 = \cos^2 t + \sin^2 t = 1$

Unit circle.

$x = \cos t \quad y = 3\sin t$ is an ellipse.

http://www.stewartcalculus.com/tec/

TEC Module 11.1A gives an animation of the relationship between motion along a parametric curve $x = f(t)$, $y = g(t)$ and motion along the graphs of f and g as functions of t. Clicking on TRIG gives you the family of parametric curves

$$x = a \cos bt \qquad y = c \sin dt$$

If you choose $a = b = c = d = 1$ and click on animate, you will see how the graphs of $x = \cos t$ and $y = \sin t$ relate to the circle in Example 2. If you choose $a = b = c = 1$, $d = 2$, you will see graphs as in Figure 8. By clicking on animate or moving the t-slider to the right, you can see from the color coding how motion along the graphs of $x = \cos t$ and $y = \sin 2t$ corresponds to motion along the parametric curve, which is called a Lissajous figure.

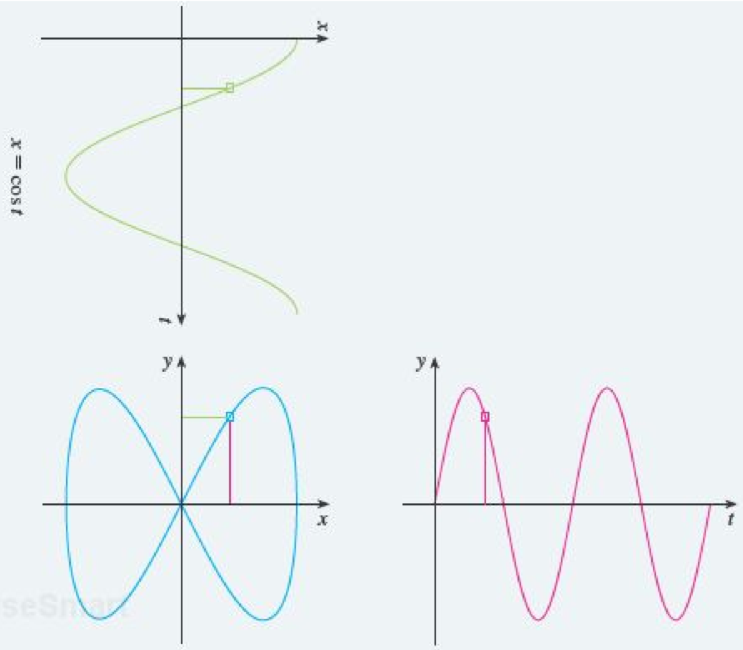


FIGURE 8 $x = \cos t \quad y = \sin 2t$ $\qquad$ $y = \sin 2t$

THE CYCLOID

EXAMPLE 7 The curve traced out by a point P on the circumference of a circle as the circle rolls along a straight line is called a **cycloid** (see Figure 13). If the circle has radius r and rolls along the x-axis and if one position of P is the origin, find parametric equations for the cycloid.

TEC An animation in Module 11.1B shows how the cycloid is formed as the circle moves.

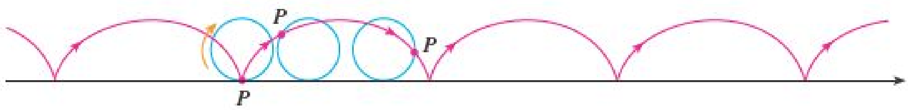


FIGURE 13

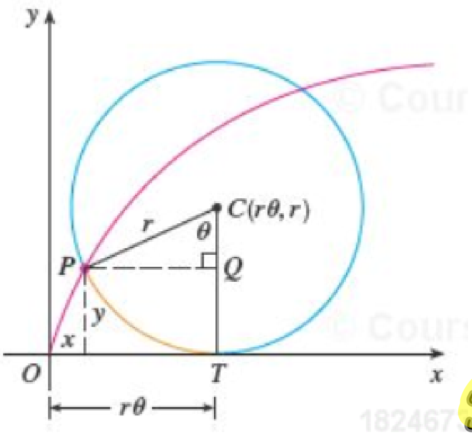
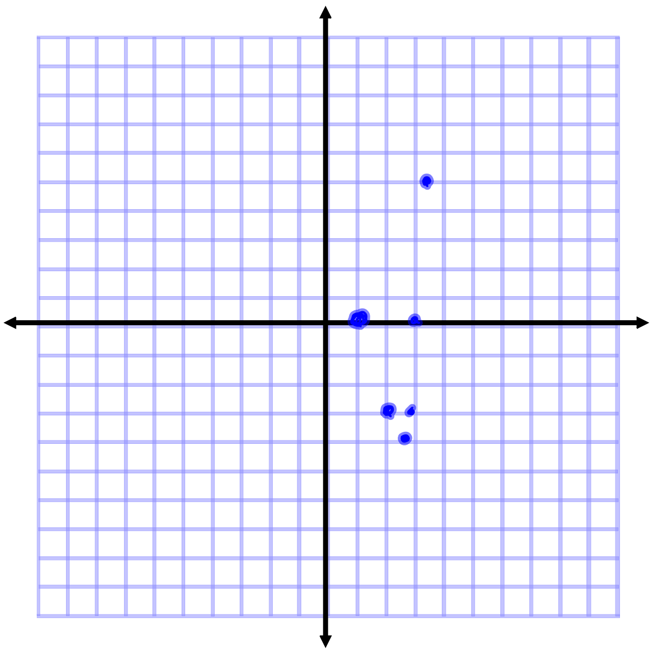


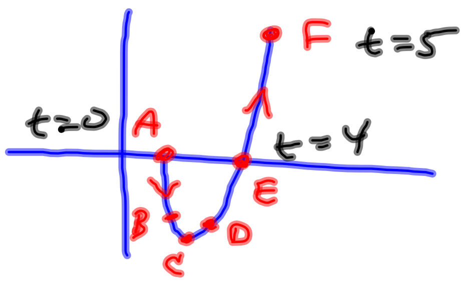
FIGURE 14

$$x = |OT| - |PQ|$$
$$= r\theta - r\sin\theta$$
$$= r(\theta - \sin\theta) = x(\theta)$$
$$y = r - |CQ|$$
$$= r - r\cos\theta$$
$$y(\theta) = r(1 - \cos\theta)$$

#1 $x = 1+\sqrt{t}$, $y = t^2 - 4t$

t	0	1	2	3	4	5
x	1	2	$1+\sqrt{2}$	$1+\sqrt{3}$	3	$1+\sqrt{5}$
	1	2	2.41	2.73	3	3.24
y	0	-3	-4	-3	0	5





A = (1,0) t=0
B = (2,-3) t=1
C = (2.41,-4) t=2
D = (2.73,-3) t=3
E = (3,0) t=4
F = (3.24,5) t=5