$$\int \tan^4 x \, dx$$

$$\int u \cos^{-1} u \, du$$ Brent is one sick puppy.

$$\int x \arccos x \, dx \qquad u = \arccos x \qquad du = -\frac{1}{\sqrt{1-x^2}} \qquad dv = x\,dx \qquad v = \frac{1}{2}x^2$$

$$uv - \int v\,du = \frac{1}{2}x^2 \arccos x + \frac{1}{2}\int \frac{x^2}{\sqrt{1-x^2}}\,dx \qquad x = \sin\theta \qquad dx = \cos\theta\, d\theta$$

$$= \frac{1}{2}x^2 \arccos x + \frac{1}{2}\int \frac{\sin^2\theta \cancel{\cos\theta}\, d\theta}{\cancel{\cos\theta}} = \frac{1}{2}x^2 \arccos x + I_1$$

$$I_1 = \frac{1}{2}\int \sin^2\theta \, d\theta = \frac{1}{2}\int \frac{1}{2}\left[1 - \cos(2\theta)\right] d\theta$$

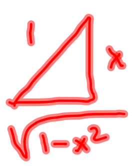
$$= \frac{1}{4}\left[\theta - \frac{1}{2}\sin(2\theta)\right] = \frac{1}{4}\theta - \frac{1}{8}\underline{\sin(2\theta)} + C$$

So, the whole ball o' wax is

$$\frac{1}{2}x^2 \arccos x + \frac{1}{4}\arcsin x - \frac{2}{8}\sin\theta\cos\theta + C$$

$$= \frac{1}{2}x^2 \arccos x + \frac{1}{4}\arcsin x - \frac{1}{4}x\sqrt{1-x^2} + C$$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$
$$= 2\sin(\arcsin x)\cos(\arcsin x)$$
$$= 2x \cdot \sqrt{1-x^2}$$, david.



$$\int \tan^4 x\,dx = \int (\sec^2 x - 1)\tan^2 x\,dx$$

$$= \int \tan^2 x \sec^2 x\,dx - \int \tan^2 x\,dx$$

$$= \int u^2\,du - \int (\sec^2 x - 1)\,dx$$

$$= \frac{u^3}{3} - \int \sec^2 x\,dx + \int dx$$

$\sin\left(\frac{\pi}{4}\right)$

$$= \frac{1}{3}\tan^3 x - \tan x + x + C$$

9 $\displaystyle\int_0^1 \frac{x\,dx}{\sqrt{x^2+4x+3}} = \int_0^1 \frac{x\,dx}{\sqrt{(x+2)^2-1}}$

x^2+4x+3
$= x^2+4x+2^2-4+3$
$= (x+2)^2-1$

Let $u = x+2$, $du = dx$
$x = u-2$
$x=0 \longrightarrow u=2$
$x=1 \longrightarrow u=3$

$\displaystyle\int_2^3 \frac{u-2}{\sqrt{u^2-1}}\,du$

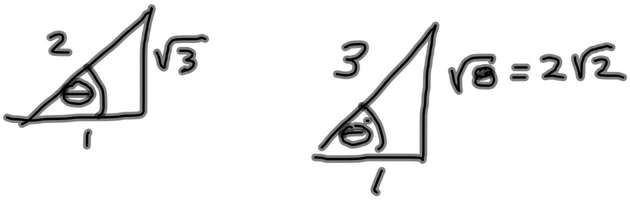
Let $u = \sec\theta$
$du = \sec\theta\tan\theta\,d\theta$
$\sqrt{u^2-1} = \tan\theta$

$u=2 \longrightarrow \theta = \sec^{-1}(2)$

$\displaystyle\int_{\sec^{-1}(2)}^{\sec^{-1}(3)} \frac{(\sec\theta-2)\sec\theta\cancel{\tan\theta}\,d\theta}{\cancel{\tan\theta}}$

$\displaystyle = \int_{\sec^{-1}(2)}^{\sec^{-1}(3)} \sec^2\theta\,d\theta - 2\int_{\sec^{-1}(2)}^{\sec^{-1}(3)} \sec\theta\,d\theta$

$\displaystyle = \tan\theta\Big]_{\sec^{-1}(2)}^{\sec^{-1}(3)} - 2\ln|\sec\theta+\tan\theta|\Big]_{\sec^{-1}(2)}^{\sec^{-1}(3)}$



$= \tan(\sec^{-1}(3)) - \tan(\sec^{-1}(2))$
$-2\Big(\ln(\sec(\sec^{-1}(3)) + \tan(\sec^{-1}(3))$
$\quad - \ln(\sec(\sec^{-1}(2)) + \tan(\sec^{-1}(2))\Big)$

$= 2\sqrt{2} - \sqrt{3} - 2\left[\ln(3+2\sqrt{2}) - \ln(2+\sqrt{3})\right]$

$\displaystyle = 2\sqrt{2} - \sqrt{3} + \ln\left(\frac{(2+\sqrt{3})^2}{(3+2\sqrt{2})^2}\right)$

$\approx .20479776\,2$

$$\int_4^\infty \frac{dx}{x^{3/4} - x^{1/2} - 1}$$

$$\boxed{\frac{1}{x^{3/4} - x^{1/2} - 1} > 0 \text{ eventually, } \forall x > 16}$$

$$\frac{1}{7} \qquad \frac{1}{7-1} \qquad \frac{1}{7-2}$$

$$\frac{1}{x^{3/4} - x^{1/2} - 1} > \frac{1}{x^{3/4}} \quad \& \quad \int_4^\infty \frac{dx}{x^{3/4}} \text{ diverges}$$

by
P-Test.

So the original must, also.
By the Comparison Test

Next Time: §11.1

Test 3: Finish by Closing Time ARC Sunday.

3 chunks. You may take 'em one at a time.

"Test 3, Test 4, Test 5"