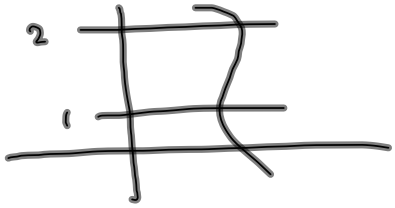
§9.1

#10 $x = \frac{y^4}{8} + \frac{1}{4y^2}$ $\quad 1 \le y \le 2$



Change in arc length

$ds = \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx \quad y=f(x)$

$= \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy \quad x=g(y)$

$\frac{dx}{dy} = \frac{1}{2}y^3 - \frac{1}{2}y^{-3}$

$\left(\frac{dx}{dy}\right)^2 = \frac{1}{4}y^6 - \frac{1}{2} + \frac{1}{4}y^{-6}$

So, $1+\left(\frac{dx}{dy}\right)^2 = \frac{1}{4}y^6 + \frac{1}{2} + \frac{1}{4}y^{-6} = \left(\frac{1}{2}y^3 + \frac{1}{2}y^{-3}\right)^2$

$= \frac{1}{4}\left[y^6 + 2 + y^{-6}\right]$

$= \frac{1}{4}y^{-6}\left[y^{12} + 2y^6 + 1\right]$ $\quad \frac{1}{2}y^3 \quad \frac{1}{2}y^{-3}$

Let $u = y^6 \Rightarrow$

$\frac{1}{4}u^{-1}\left[u^2 + 2u + 1\right]$

$= \frac{1}{4}u^{-1}\left[u+1\right]^2$

$\sqrt{3^2} = 3$

$\sqrt{(-3)^2} = 3$

$\therefore \int_1^2 \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy = \int_1^2 \sqrt{\left(\frac{1}{2}y^3 + \frac{1}{2}y^{-3}\right)^2}\,dy$

$= \int_1^2 \left|\frac{1}{2}y^3 + \frac{1}{2}y^{-3}\right| dy = \int_1^2 \left(\frac{1}{2}y^3 + \frac{1}{2}y^{-3}\right) dy$ *

Because $y \in [1,2] \Rightarrow y^3 \& y^{-3} > 0$

$= \left[\frac{1}{8}y^4 - \frac{1}{4}y^{-2}\right]_1^2$

$= 2 - \frac{1}{16} - \left(\frac{1}{8} - \frac{1}{4}\right)$

$= \frac{32-1}{16} + \frac{2}{16} = \frac{33}{16}$

$\sqrt{x^2} = |x|$

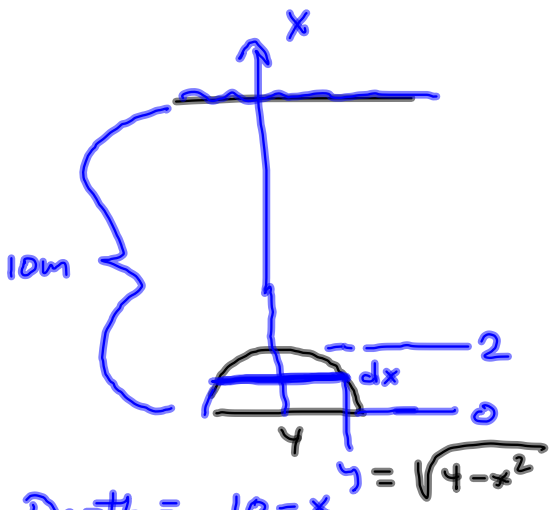
$(\sqrt{x})^2 =$

$(x^2)^{\frac{1}{2}} = (x^{\frac{1}{2}})^2$

$x^{\frac{3}{2}} = (x^{\frac{1}{2}})^3 = (x^3)^{\frac{1}{2}}$

provided $x \geq 0$

$\sqrt[3]{x^2} = (\sqrt[3]{x})^2$



Force on semicircular gate @ bottom of dam

Depth = $10 - x$

width = $2y = 2\sqrt{4-x^2}$

$x^2 + y^2 = 2^2$

Force $\int_0^2 2\rho g (10-x)\sqrt{4-x^2}\, dx$

$= 2\rho g \left[\int_0^2 10\sqrt{4-x^2}\, dx + \frac{1}{2}\int_0^2 -2x\sqrt{4-x^2}\, dx \right]$

$= 20\rho g \cdot 2\pi + \frac{1}{2}\int_4^0 u^{\frac{1}{2}} du \cdot 2\rho g$

$u = 4-x^2$

$du = -2x\,dx$

$x = 0 \Rightarrow u = 4$

$x = 2 \Rightarrow u = 0$

$= 40\rho g\pi + \frac{1}{2}\left[\frac{2}{3}u^{3/2}\right]_4^0 \cdot 2\rho g$

$= 40\rho g\pi - \frac{1}{2}\left[\frac{2}{3}u^{\frac{3}{2}}\right]_0^4 \cdot 2\rho g$

$= 40\rho g\pi - \frac{2}{3}\rho g[8] = 40\rho g\pi - \frac{16\rho g}{3}$

$20\rho g\pi - \frac{16\rho g}{3}$