Nick sez:

One of my girlfriend's kids.
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§9.2 #18 Surface Area of Surfaces of Revolution

$$S = \int 2\pi x \, ds \quad \text{OR} \quad \int 2\pi y \, ds$$

y-axis: x-axis:

$$2\pi \int x \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx \qquad 2\pi \int f(x) \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$$

$$2\pi \int y \sqrt{1 + \left(\frac{dx}{dy}\right)^2} \, dy \qquad 2\pi \int y \sqrt{1 + \left(\frac{dx}{dy}\right)^2} \, dy$$

$x = g(y)$ version $x = g(y)$ version

(18) $y = x + \sqrt{x}$, $1 \le x \le 2$, about x-axis

$$y' = 1 + \frac{1}{2\sqrt{x}} \Longrightarrow (y')^2 = 1 + \frac{1}{\sqrt{x}} + \frac{1}{4x}$$

$$1 + (y')^2 = 2 + \frac{1}{\sqrt{x}} + \frac{1}{4x}$$

$$\therefore \; S.A. = 2\pi \int_1^2 (x + \sqrt{x}) \sqrt{2 + \frac{1}{\sqrt{x}} + \frac{1}{4x}} \, dx$$

Evaluate using Simpson's Rule, $n = 10$

A	B	C	D	E
		f(x)		
0	1	3.605551	0	3.605551
1	1.1	3.832318	1	15.32927
2	1.2	4.055347	0	8.110694
3	1.3	4.275091	1	17.10036
4	1.4	4.491919	0	8.983838
5	1.5	4.706135	1	18.82454
6	1.6	4.917993	0	9.835986
7	1.7	5.12771	1	20.51084
8	1.8	5.335469	0	10.67094
9	1.9	5.541431	1	22.16572
10	2	5.745734	0	5.745734
				140.8835
				4.696116
		Surface Area:		29.50657

Sweet!

x^(1/2)

sqrt(x)

where x = a specific cell ref.

multiply by 2π.

$S.A. \approx 29.50656629$

See website for this.

§10.3 #48

Show that $v_0 = \sqrt{\frac{2gRh}{R+h}}$ where h is max height.

We're trying to solve for max height.

$\frac{dv}{dt} = 0$

x = height above surface

R = radius of Earth.

g = acceleration of gravity.

$F = \frac{mgR^2}{(x+R)^2}$

$m\frac{dv}{dt} = \frac{mgR^2}{(x+R)^2}$

$\frac{dv}{dt} = -\frac{gR^2}{(x+R)^2}$

But $x = x(t)$ is a function. I treated it as a constant. BAD!

$dv = -\frac{gR^2}{(x+R)^2}\,dt$

$v = -\frac{gR^2}{(x+R)^2}t + C = -\frac{gR^2}{(x+R)^2}t + V_0$

$V_0 = v(0) = C = V_0$

Set $v(t) = 0$ is max height achieved.

$$\Rightarrow v(t) = -\frac{gR^2}{(x+R)^2} t + v_0 = 0$$

$$\Rightarrow \quad v_0 = \frac{gR^2}{(x+R)^2} t$$

$$m \frac{dv}{dt} = m v \frac{dv}{dx}$$

$$\frac{dv}{dt} = -\frac{gR^2}{(x+R)^2} \qquad v \frac{dv}{dx} = -\frac{gR^2}{(x+R)^2}$$

$$\int v \, dv = \int \frac{-gR^2}{(x+R)^2} dx$$

$$\frac{1}{2} v^2 = \frac{gR^2}{(x+R)} + C$$

@ max height, $v = 0$

$$\frac{gR^2}{x+R} + C = 0 \quad ?$$

$$\frac{1}{2}v_0^2 = \frac{gR^2}{x+R} + C \qquad x=0 \text{ @ } t=0$$

$$\frac{1}{2}v_0^2 = \frac{gR^2}{R} + C = gR + C$$

$$C = \frac{1}{2}v_0^2 - gR$$

$$\frac{1}{2}v^2 = \frac{gR^2}{(x+R)} + \frac{1}{2}v_0^2 - gR$$

Experimenting

when max height is achieved, we have

$$0 = \frac{gR^2}{h+R} + \frac{1}{2}v_0^2 - gR$$

$$-\frac{1}{2}v_0^2 = -gR + \frac{gR^2}{h+R} = \frac{-gRh - gR^2 + gR^2}{h+R}$$

$$-\frac{1}{2}v_0^2 = \frac{-gRh}{h+R}$$

$$v_0^2 = \frac{2gRh}{h+R}$$

$$v_0 = \pm\sqrt{\frac{2gRh}{h+R}} \qquad \& \; v_0 > 0 \implies v_0 = +\sqrt{\frac{2gRh}{h+R}}$$