

$\int 10.5$ #s 1-6, 8, 13, 17, 20, 23, 24, 26*

* 26 is a 2nd-order equation

Test Monday Tomorrow & Sunday Review
 $\int 9.3$ OMIT. Put it on Final.

AMPERSAND

$\int 10.5$ Linear Differential Equations

$$(xy)' = \frac{d}{dx}[xy]$$

$$\nexists \int (xy)' dx = \int \frac{d}{dx}[xy] dx = \int d(xy) = xy + C$$

$$\frac{d}{dx}[xy] = y + xy'$$

$$\text{consider } y' + \frac{1}{x}y = 2$$

$$\underbrace{xy'}_{} + y = 2x$$

$$(xy)'$$

$$(xy)' = 2x$$

$$\int (xy)' dx = \int 2x dx$$

$$xy = x^2 + C$$

$$y = x + \frac{C}{x}$$

Multiplying both sides by x made the left hand side into a recognizable derivative.

That allowed us to integrate to a solution.

" x " was our integrating factor.

Linear D.E.:

$$y' + P(x)y = Q(x)$$

Want $I(x)$ so that

$$I(x)[y' + P(x)y] = I(x)Q(x)$$

$(I(x)y)'$

If we have such an $I(x)$, then

$$I(x)y = \int (I(x)y)' dx = \int I(x)Q(x) dx$$

$$y = \frac{\int I(x)Q(x) dx}{I(x)}$$

Let's look at $I(x)[y' + P(x)y]$

$$= I(x)y' + I(x)P(x)y = (I(x)y)' = I'(x)y + I(x)y'$$

$$I(x)P(x)y = I'(x)y$$

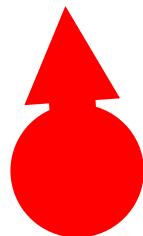
$$I(x)P(x) = I'(x)$$

$$e^{\int P(x) dx} = \frac{I'(x)}{I(x)} dx = \ln |I(x)|$$

$$e^{\int P(x) dx} = |I(x)|$$

Let $I(x) = e^{\int P(x) dx}$
is the integrating factor

for $y' + P(x)y = Q(x)$



$$\textcircled{5} \quad y' + 2y = 2e^x \quad \leftarrow \quad P(x) = 2 \quad e^{\int 2dx} = e^{2x+C} \quad \text{Ditch it.}$$

$$e^{2x} [y' + 2y] = e^{2x} \cdot 2e^x$$

$$\int \frac{d}{dx} [e^{2x} y] dx = \int 2e^{3x} dx$$

$$ye^{2x} = \frac{2}{3}e^{3x} + C$$

$$y = \frac{2}{3}e^x + Ce^{-2x}$$

Check: $y' + 2y$

$$= \frac{2}{3}e^x - 2Ce^{-2x} + 2\left(\frac{2}{3}e^x + Ce^{-2x}\right)$$

$$= \frac{2}{3}e^x - 2Ce^{-2x} + \frac{4}{3}e^x + 2Ce^{-2x}$$

$$= \left(\frac{2}{3} + \frac{4}{3}\right)e^x = \frac{6}{3}e^x = 2e^x$$

(11)

$$\sin x \cdot y' + \cos x \cdot y = \sin(x^2)$$

$$y' + P(x)y = Q(x)$$

$$\sin x \cdot y' + \cos x \cdot y = \int (\sin x \cdot y)' dx = \int \sin(x^2) dx$$

$$\sin x \cdot y = \int \sin(x^2) dx$$

$u = x^2 \quad du = 2x dx$
leads nowhere.

$$y = \frac{\int \sin(x^2) dx}{\sin x}$$

$$y' + \cot x \cdot y = \frac{\sin(xy)}{\sin x}$$

$$e^{\int \cot x dx} = e^{\ln |\sin x|} = |\sin x|$$

$$\int \frac{\cos \sin x}{\sin x}$$

$= \sin x$ w/appropriate
restrictions.