§10.5 #s 1–6, 8, 13, 17, 20, 23, 24, 26*

* 26 is a 2nd-order equation

Test Monday Tomorrow & Friday Review

§9.3 OMIT. Put it on Final.

§10.5 Linear Differential Equations

AMPERSAND

$$(xy)' = \frac{d}{dx}[xy]$$

$$\& \int (xy)'\,dx = \int \frac{d}{dx}[xy]\,dx = \int d(xy) = xy + C$$

$$\frac{d}{dx}[xy] = y + xy'$$

Consider $y' + \frac{1}{x}y = 2$

$$\underbrace{xy' + y}_{(xy)'} = 2x$$

$$(xy)' = 2x$$

$$\int (xy)'\,dx = \int 2x\,dx$$

$$xy = x^2 + C$$

$$y = x + \frac{C}{x}$$

Multiplying both sides by x made the left hand side into a recognizable derivative. That allowed us to integrate to a solution. "x" was our integrating factor.

Linear D.E.:

$$y' + P(x)y = Q(x)$$

Want $I(x)$ so that

$$\underbrace{I(x)\left[y' + P(x)y\right]}_{(I(x)y)'} = I(x)Q(x)$$

If we have such an $I(x)$, then

$$I(x)y = \int (I(x)y)'\,dx = \int I(x)Q(x)\,dx$$

$$y = \frac{\int I(x)Q(x)\,dx}{I(x)}$$

Let's look @ $I(x)\left[y' + P(x)y\right]$

$$= I(x)y' + I(x)P(x)y = (I(x)y)' = I'(x)y + I(x)y'$$

$$I(x)P(x)y = I'(x)y$$

$$I(x)P(x) = I'(x)$$

$$e^{\int P(x)\,dx = \int \frac{I'(x)}{I(x)}\,dx} = e^{\ln|I(x)|}$$

$$e^{\int P(x)dx} = |I(x)|$$

Let $I(x) = e^{\int P(x)dx}$ is the integrating factor for $y' + P(x)y = Q(x)$

5 $y' + 2y = 2e^{x}$

$P(x) = 2$

$e^{\int 2dx} = e^{2x+C}$ Ditch it.

$e^{2x}\left[y' + 2y\right] = e^{2x} \cdot 2e^{x}$

$\int \frac{d}{dx}\left[e^{2x}y\right]dx = \int 2e^{3x}dx$

$e^{\int p(x)dx}$

$ye^{2x} = \frac{2}{3}e^{3x} + C$

$y = \frac{2}{3}e^{x} + Ce^{-2x}$

Check: $y' + 2y$

$= \frac{2}{3}e^{x} - 2Ce^{-2x} + 2\left(\frac{2}{3}e^{x} + Ce^{-2x}\right)$

$= \frac{2}{3}e^{x} - 2Ce^{-2x} + \frac{4}{3}e^{x} + 2Ce^{-2x}$

$= \left(\frac{2}{3} + \frac{4}{3}\right)e^{x} = \frac{6}{3}e^{x} = 2e^{x}$

11 $\sin x \, y' + \cos x \, y = \sin(x^2)$

$y' + P(x) y = Q(x)$

$\sin x \, y' + \cos x \, y = \int (\sin x \, y)' dx = \int \sin(x^2) dx$

$\sin x \, y = \int \sin(x^2) dx$

$u = x^2 \quad du = 2x \, dx$
leads NOwhere.

$$\boxed{y = \frac{\int \sin(x^2) dx}{\sin x}}$$

$y' + \cot x \, y = \frac{\sin(x^2)}{\sin x}$

$e^{\int \cot x \, dx} = e^{\ln|\sin x|} = |\sin x|$

$\int \frac{\cos x}{\sin x}$

$= \sin x$ w/ appropriate restrictions.