§10.3 #s 1, 4, 7, 10, 13, 18, 21, 44, 48

16

$$\boxed{y' = x(y^2-4)}$$ Use Direction Field Grapher for this.

is separable

$$\frac{dy}{dx} = x(y^2-4)$$

$$\int \frac{dy}{y^2-4} = \int x\,dx \qquad \frac{A}{y-2} + \frac{B}{y+2} = \frac{1}{4}\cdot\frac{1}{y-2} - \frac{1}{4}\cdot\frac{1}{y+2}$$

$$\frac{1}{4}\ln(y-2) - \frac{1}{4}\ln(y+2) = \frac{1}{2}x^2 + C^*$$

$$\ln(y-2) - \ln(y+2) = 2x^2 + 4C^*$$

$$e^{\ln\left(\frac{y-2}{y+2}\right)} = e^{2x^2+4C^*}$$

$$\frac{y-2}{y+2} = e^{2x^2+4C^*} = e^{2x^2}\,e^{4C^*}$$

C

$$\frac{y-2}{y+2} = Ce^{2x^2}$$

$$y-2 = (y+2)Ce^{2x^2} = yCe^{2x^2} + 2Ce^{2x^2}$$

$$y - yCe^{2x^2} = 2Ce^{2x^2} + 2$$

$$y(1 - Ce^{2x^2}) = 2Ce^{2x^2} + 2$$

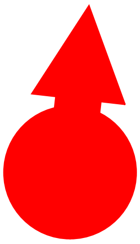
$$y = \frac{2Ce^{2x^2}+2}{1-Ce^{2x^2}}$$

10.3 Separable Equations

Recall

$y' = x(y^2-4)$ from Monday

(Included in today's notes).



$\boxed{E1}$ $\dfrac{dy}{dx} = \dfrac{x^2}{y^2}$

$\int y^2\,dy = \int x^2\,dx$ is the theme.

A "key step"

#21 §10.3 $y' = x+y$

Let $u = x+y$

$\Rightarrow u' = 1 + y'$ if we differentiate with respect to x.

So, $y' = u' - 1$

$u' - 1 = u$

$u' = u + 1$

Key step $\rightarrow$ $u' = \dfrac{du}{dx} = u+1$

$\Rightarrow \int \dfrac{du}{u+1} = \int dx$ Divide by u+1, here!

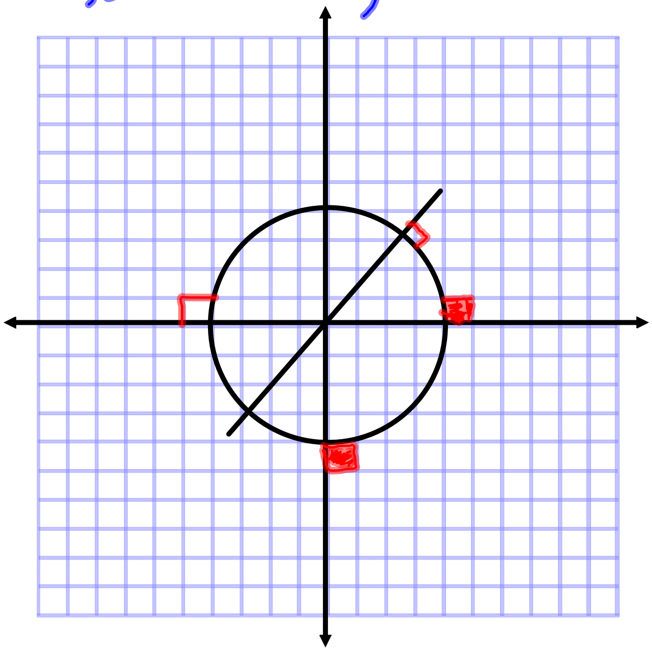
$u' - u = 1$

$\int (u' - u) = \int 1$?

Leads nowhere.

Orthogonal Trajectories

The family $\{ y = mx \mid m \in \mathbb{R} \}$ is orthogonal to the family of circles $\{ x^2 + y^2 = r^2 \mid r \in \mathbb{R} \}$



-0 E5

$x = ky^2$ are parabolas

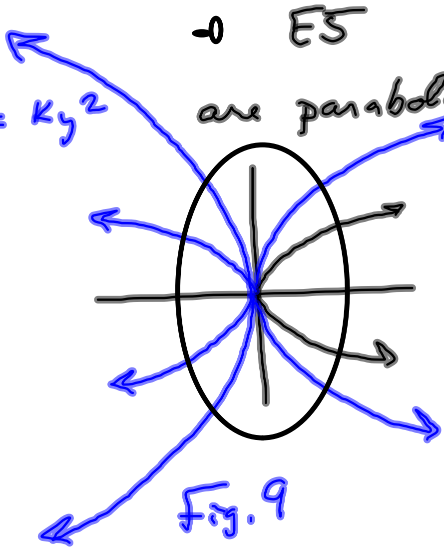


Fig. 9

Orthogonal Family is ellipses centered @ the origin.

Differentiate:

$1 = 2kyy'$

Chain Rule: $\frac{d}{dx}[y^2] = 2y \cdot \frac{dy}{dx} = 2yy'$

$y' = \frac{1}{2ky}$

$x = ky^2$

$\Longrightarrow \frac{x}{y^2} = k$

$= \frac{1}{2\left(\frac{x}{y^2}\right)y}$

$= \frac{1}{2\frac{x}{y}} = \frac{y}{2x}$

More important,

$y' = \frac{y}{2x} \Longrightarrow$

orthogonal family satisfies

$y' = \frac{y}{2x}$ is a separable ~~form~~ eq'n.

$y' = -\frac{2x}{y}$

d

$m_\perp = -\frac{1}{m}$

$$y' = -\frac{2x}{y}$$

$$\frac{dy}{dx} = -\frac{2x}{y}$$

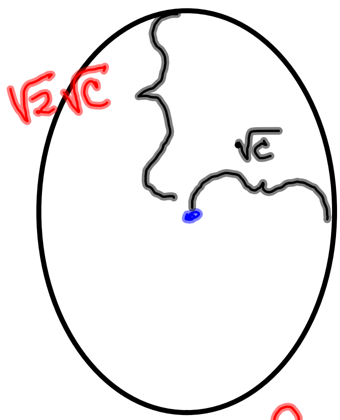
$$\int y\,dy = \int -2x\,dx$$

$$\frac{1}{2}y^2 = -x^2 + C$$

$$x^2 + \frac{y^2}{2} = C$$

$$\frac{x^2}{c} + \frac{y^2}{2c} = 1$$

$$\frac{x^2}{(\sqrt{c})^2} + \frac{y^2}{(\sqrt{2}\sqrt{c})^2} = 1$$



Steve forgot

Mixing Probs

y = the amt. of salt (in Kg) in the vat
as a function of
t = time (in min.)

$\exists$ 20 Kg in 5000 L of water to start.

A mix of .03 Kg salt per L is pumped in
@ 25 L/min. Drains @ 25 L/min.

Assume perfect mixing.

Find $y(30)$

rate in: $\left(.03 \frac{Kg}{L}\right)\left(25 \frac{L}{min}\right) = .\frac{75\ Kg}{min}$

rate out $\left(\frac{y(t)\ Kg}{5000\ L}\right)\left(25 \frac{L}{min}\right) = \frac{y(t)}{200} \frac{Kg}{min}$

So, $\frac{dy}{dt} = y'(t) =$ rate in $-$ rate out

$$= .75 - \frac{1}{200} y(t)$$

$$= \frac{75}{100} - \frac{1}{200} y(t)$$

$$y' = \frac{150 - y(t)}{200} = \frac{dy}{dt}$$

$$\therefore \frac{dy}{150-y} = \frac{1}{200} dt$$

$$\int \frac{dy}{150-y} = \int \frac{1}{200}\,dt$$

$$-\ln|150-y| = \frac{1}{200}t + C$$

Part ways with the book.

$$\ln|150-y| = -\frac{1}{200}t - C$$

$$|150-y| = e^{-\frac{1}{200}t - C} = e^{-\frac{1}{200}t}e^{-C} = Ae^{-\frac{1}{200}t}$$

I solved this: * Read book explanation

$$150-y = Ae^{-\frac{1}{200}t}$$

$$-y = Ae^{-\frac{1}{200}t} - 150$$

$$y(t) = 150 - Ae^{-\frac{1}{200}t}$$

$$y(0) = 150 - Ae^{0} = 20 = 150 - A$$

$$\Longrightarrow A = 150 - 20 = 130; \; \circ \; \circ \; \circ,$$

$$y(t) = 150 - 130e^{-\frac{1}{200}t}$$

I didn't account for "| |"