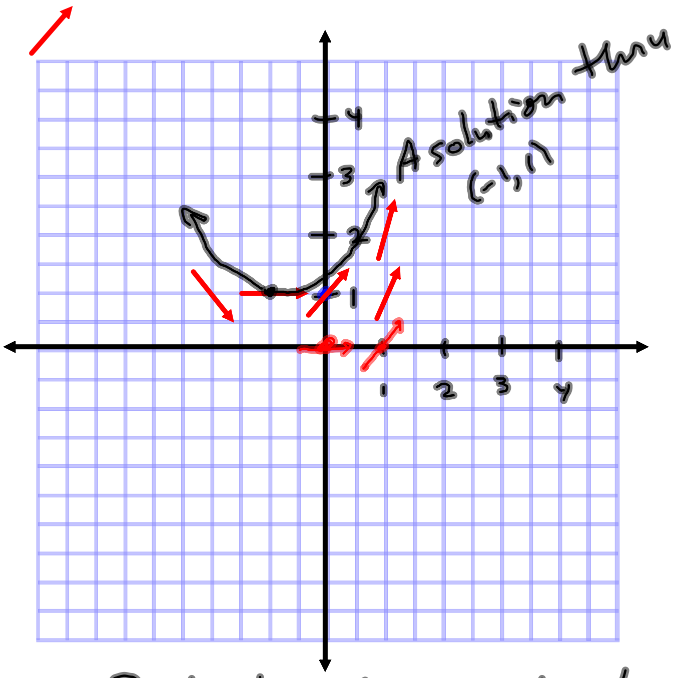
§10.2 Direction Fields & Euler's Method



Pg 608

$y' = x + y \qquad y(0) = 1$

$(0,1): y' = 0+1 = 1$

$(1,1): y' = 1+1 = 2$

$(1,2): y' = 3$

$(-1,1): y' = 0$

$(-2,1): y' = -1$

See Figure 3

Picked $y(-1) = 1$ to be an initial value.

Due Friday: 1ai, aii, 6; 2ai, aii, b; 3-7; 10, 11, 16, 19, 20, 25

$y' = x + y$. $y(0) = 1$. Use Euler's Method.

$(0,1)$: $0 + 1 = 1 = y' = m_{tan} = F(x,y)$

$y_1 = y_0 + m(x - x_0)$ ← Point-Slope of line thru (x_0, y_0) with slope m

$= y_0 + F(x_0, y_0)(x - x_0)$

This is the first piece of our Euler Solution:

$y_1 = 1 + 1(x - 0)$

Now, each $x - x_0$ OR $x_1 - x_0$ OR $x_2 - x_1$ will be $\Delta x = h$

Use steps of length $h = .2$ to build an Euler meth. soln for $y(1) = ?$

$(x_0, y_0) = (0, 1)$

$y_1 = y_0 + h \cdot F(0,1)$

$= 1 + .2 \cdot 1$

$= 1.2$

$y_2 = y_1 + h \cdot F(.2, 1.2)$

$= 1.2 + .2 \cdot 1.4$ Slope is steeper & steeper.

$= 1.2 + .28$

$= 1.48$

$y_3 = y_2 + h \cdot F(.4, 1.48)$

$= 1.48 + .2 \cdot 1.88$

$= 1.48 + .376$

$= 1.856$, etc. I'm tired

Book way

n	x	y
0	0	1
1	.2	1.2
2	.4	1.48
3	.6	1.856
4	.8	
5	1.0	$\boxed{\text{ANSWER}}$ → y(1)
6		

$F(.4, 1.48) = .4 + 1.48$

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$$\boxed{y' = x(y^2-4)}$$ Use Direction Field Graphes for this.

is separable

$$\frac{dy}{dx} = x\,(y^2-4)$$

$$\int \frac{dy}{y^2-4} = \int x\,dx \qquad \frac{A}{y-2} + \frac{B}{y+2} = \frac{1}{4}\cdot\frac{1}{y-2} - \frac{1}{4}\cdot\frac{1}{y+2}$$

$$\frac{1}{4}\ln(y-2) - \frac{1}{4}\ln(y+2) = \frac{1}{2}x^2 + c^*$$

$$\ln(y-2) - \ln(y+2) = 2x^2 + 4c^*$$

$$e^{\ln\left(\frac{y-2}{y+2}\right)} = e^{2x^2+4c^*}$$

$$\frac{y-2}{y+2} = e^{2x^2+4c^*} = e^{2x^2}\cdot \underbrace{e^{4c^*}}_{C}$$

$$\frac{y-2}{y+2} = Ce^{2x^2}$$

$$y-2 = (y+2)Ce^{2x^2} = yCe^{2x^2} + 2Ce^{2x^2}$$

$$y - yCe^{2x^2} = 2Ce^{2x^2} + 2$$

$$y(1-Ce^{2x^2}) = 2Ce^{2x^2} + 2$$

$$y = \frac{2Ce^{2x^2}+2}{1-Ce^{2x^2}}$$