10.1 Modeling with Differential Equations

A differential equation is an equation that relates a function and one or more of its derivatives.

Concepts

- diff eq.; order; degree; family of solutions
- initial condition(s); solutions to initial value problems

A differential equation is an equation involving $x, y', y'', \ldots, y^{(n)}$ where y is a function of x with n derivatives.

The n in the highest $y^{(n)}$ is the order of the equation and the degree is the exponent that $y^{(n)}$ has

$$2(y''')^4 + 5xy' + 7x = 0$$

3rd order degree of 4

A particular solution to a diff. eq. is a function $y = f(x)$ that satisfies the diff. eq.

A general solution is an expression with arbitrary constants that represent the family of all particular solutions.

We know how to solve a diff. eq. of the form $y' = f(x)$ by integration $y = \int f(x)\,dx$

like
4

ex a) For what values of r does the function $y = e^{rt}$ satisfy $y'' + y' - 6y = 0$?

$y = e^{rt}$

$y' = re^{rt}$

$y'' = r^2e^{rt}$

$y'' + y' - 6y = 0$

$r^2e^{rt} + re^{rt} - 6e^{rt} = 0$

$e^{rt}(r^2 + r - 6) = 0$

$e^{rt}(r+3)(r-2) = 0$

$\underbrace{e^{rt} = 0}_{\text{undef. no sol.}}$ $\quad r + 3 = 0 \quad r - 2 = 0$

$\underline{\underline{r = -3}} \quad \underline{\underline{r = 2}}$

Initial boundary conditions are specified function values that are used to determine a particular solution to a diff eq. from the general solution.

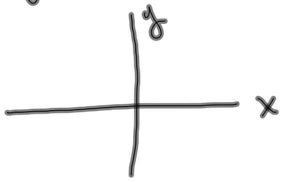
like 6 ex b)

a) Show that every member of the family of functions, $y=Ce^{x^2/2}$ is a solution to the diff eq. $y' = xy$

$y = Ce^{x^2/2}$ | $y' = xy$
$y' = C\frac{2x}{2}e^{x^2/2}$ | $y' = x(Ce^{x^2/2})$
$\underline{\underline{y' = Cxe^{x^2/2}}}$ | $\underline{\underline{y' = Cxe^{x^2/2}}}$

same

b) Graph several members of the family on the same set of axes (common screen)



$y = 2xe^{x^2/2}$

$y = xe^{x^2/2}$

$y = -3xe^{x^2/2}$

etc...

c) Find a solution to the diff. eq. $y' = xy$ that satisfies the initial condition $y(0) = 5$

$y = Ce^{x^2/2}$ $\quad y(0) = 5 \quad (0,5)$

solve for $\underline{\underline{C}}$

$5 = Ce^{0/2}$

$5 = C \quad y = 5e^{x^2/2}$

d) with initial condition $y(1) = 2$

$y = Ce^{x^2/2} \quad y(1) = 2$

$2 = Ce^{1/2} \quad y = 2e^{-1/2} \cdot e^{x^2/2}$

$C = 2e^{-1/2} \quad y = 2e^{\frac{x^2-1}{2}}$

10) $y(t)$ satisfies

$$\frac{dy}{dt} = y^4 - 6y^3 + 5y^2$$

a) $y = k$ $\qquad 0 = k^4 - 6k^3 + 5k^2$

$y' = 0$ $\qquad k^2(k^2 - 6k + 5) = 0$

$\uparrow$ $\qquad k^2(k-5)(k-1) = 0$

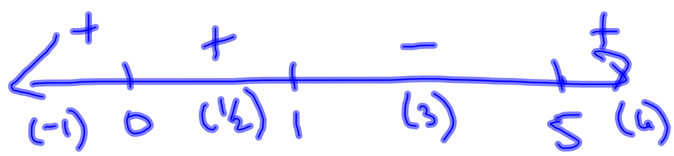
$\frac{dy}{dt}$ $\qquad k^2 = 0 \quad \underline{\underline{k=5}} \quad \underline{\underline{k=1}}$

$\underline{\underline{k=0}}$

b) y increasing

$\frac{dy}{dt} > 0$ $\qquad y^4 - 6y^3 + 5y^2 > 0$

$y^2(y-5)(y-1) > 0$



interval notation $y \in (-\infty, 0) \cup (0, 1) \cup (5, \infty)$

c) y decreasing

$y \in (1, 5)$

Homework for 10.1

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