9.4 - Applications to Economics and Biology

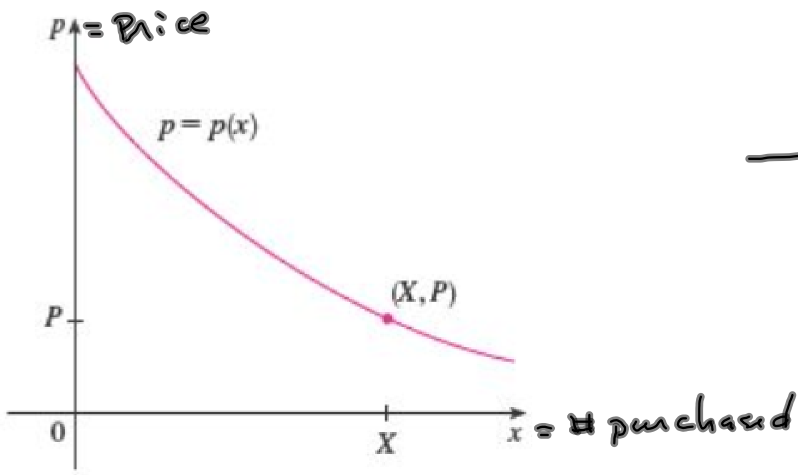
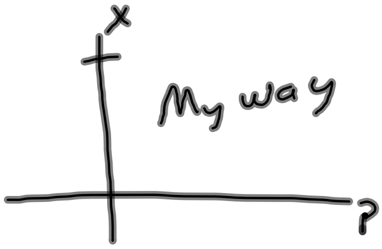


FIGURE 1
A typical demand curve



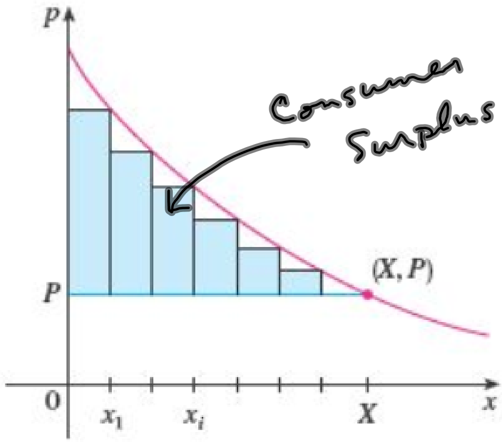
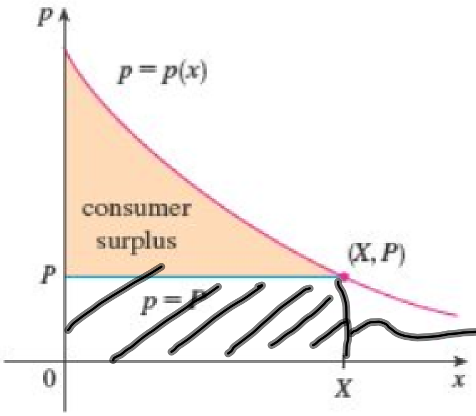


FIGURE 2



$$C.S. = \int_0^X (p(x) - P)\,dx$$

$$PX = \text{Area} = \int_0^X P\,dx$$

FIGURE 3

The consumer surplus represents the amount of money saved by consumers in purchasing the commodity at price P, corresponding to an amount demanded of X. Figure 3 shows the interpretation of the consumer surplus as the area under the demand curve and above the line $p = P$.

Example - The demand function for a certain commodity is p = 1500 - 0.03x^2. Find the consumer surplus when the sales level is 200. Illustrate by drawing the demand curve and identifying the consumer surplus as an area.

$p(x) = 1500 - .03x^2$

$x = 200 \Longrightarrow p(x) = p(200) = 1500 - .03(200)^2$
$= 1500 - 1200$
$= 300 = P$

$\int_0^{200} P\,dx$

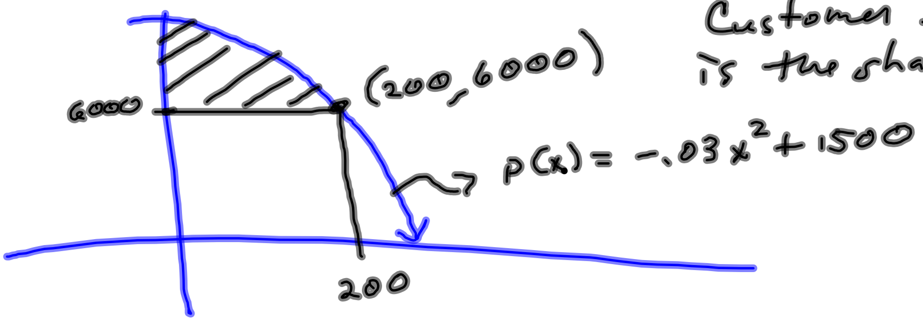
Consumer Surplus
$= \int_0^{200} (p(x) - P)\,dx = \int_0^{200} (1500 - .03x^2)\,dx - (300)(200)$

$= \left[1500x - .01x^3\right]_0^{200} - 60000$

$= 1500(200) - .01(200)^3 - 0 - 6000$

$= 300000 - 80000 - 6000$

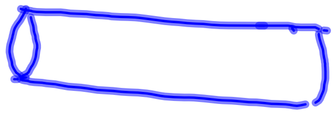
$= 220000 - 6000 = \$214,000$ is C.S.



Customer Surplus is the shaded area.

BLOOD FLOW

$$v(r) = \frac{P}{4\eta l}(R^2 - r^2)$$



v = the velocity of blood
R = radius of blood vessel
l = length of the blood vessel
r = distance from the center of the vessel
P = pressure difference between the ends of the vessel
η = viscosity of the blood eta

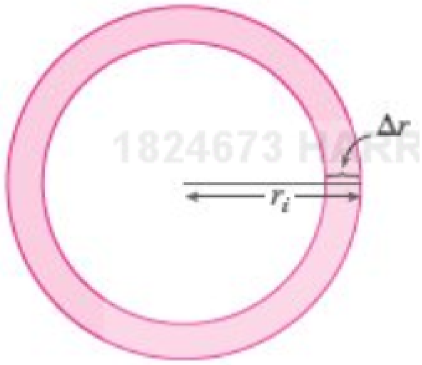


FIGURE 4

"Flux" means "flow," which is volume per unit time.

Let r_1, r_2, r_3, ... be smaller, equally spaced radii

Area of ring i is given by

$$2\pi r_i \, \Delta r \qquad \text{where} \quad \Delta r = r_i - r_{i-1}$$

So the volume of blood through ring i *per unit time* is given by

$$(2\pi r_i \, \Delta r)\, v(r_i) = 2\pi r_i \, v(r_i)\, \Delta r$$

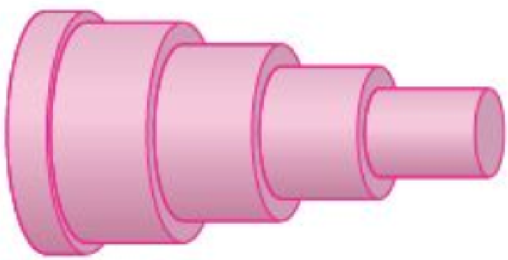


FIGURE 5

Figure 5 is a picture of how this would look in unit time. As you get closer to the center, the velocity increases, which is why this looks like a telescope.

So the flux F in unit time is given by

$$F = \lim_{n\to\infty} \sum_{i=1}^{n} 2\pi r_i\, v(r_i)\, \Delta r = \int_0^R 2\pi r\, v(r)\, dr$$

$$v(r) = \frac{P}{4\eta l}(R^2 - r^2)$$

$$= \int_0^R 2\pi r \frac{P}{4\eta l}(R^2 - r^2)\, dr$$

$$= \frac{\pi P}{2\eta l}\int_0^R (R^2 r - r^3)\, dr = \frac{\pi P}{2\eta l}\left[R^2 \frac{r^2}{2} - \frac{r^4}{4}\right]_{r=0}^{r=R}$$

$$= \frac{\pi P}{2\eta l}\left[\frac{R^4}{2} - \frac{R^4}{4}\right] = \frac{\pi P R^4}{8\eta l}$$

<u>Pouiselle's Law</u>

$$F = \frac{\pi P R^4}{8\eta l}$$

Cardiac Output

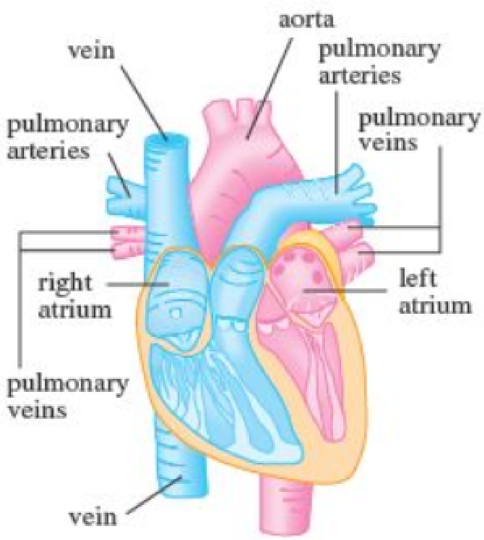


FIGURE 6

The *dye dilution method* is used to measure the cardiac output. Dye is injected into the right atrium and flows through the heart into the aorta. A probe inserted into the aorta measures the concentration of the dye leaving the heart at equally spaced times over a time interval $[0, T]$ until the dye has cleared. Let $c(t)$ be the concentration of the dye at time t. If we divide $[0, T]$ into subintervals of equal length Δt, then the amount of dye that flows past the measuring point during the subinterval from $t = t_{i-1}$ to $t = t_i$ is approximately

$$(\text{concentration})(\text{volume}) = c(t_i)(F\,\Delta t)$$

where F is the flux, or blood flow, as before. And, as before, it's the one unknown in this situation.

$$\sum_{i=1}^{n} c(t_i)F\,\Delta t = F\sum_{i=1}^{n} c(t_i)\,\Delta t$$

$$A = F\int_0^T c(t)\,dt$$

Can you think of an unstated assumption that goes into this model?

$$F = \frac{A}{\int_0^T c(t)\,dt}$$

F is constant

T is big enough $\ni$ all the dye is pumped thru.

Then: $\int_0^{T+37} c(t)\,dt = \int_0^T c(t)\,dt$

17. The dye dilution method is used to measure cardiac output with 6 mg of dye. The dye concentrations, in mg/L, are modeled by $c(t) = 20te^{-0.6t}$, $0 \leq t \leq 10$, where t is measured in seconds. Find the cardiac output.

$$= F = \frac{A}{\int_0^T c(t)\,dt} \quad \frac{mg}{\quad}$$

$T = 10$

$A = 6$

$c(t) = 20te^{-.6t}$

So, $\int_0^T c(t)\,dt = \int_0^{10} 20te^{-.6t}\,dt$

$$= 20\int_0^{10} te^{-0.6t}\,dt$$

$c(t)$ mg/L-s ?

Scratch

$u = t \Rightarrow du = dt$

$dv = e^{-0.6t}\,dt \Rightarrow \frac{1}{-0.6}e^{-0.6t} = v$

So $uv - \int v\,du = t \cdot \left(-\frac{1}{0.6}e^{-.6t}\right) - \int \frac{1}{-0.6}e^{-0.6t}\,dt$

$$= -\frac{t}{0.6}e^{-.6t} + \frac{1}{0.6} \cdot \left(-\frac{1}{0.6}\right)e^{-.6t}$$

Evaluate for 0 to 10 : Don't forget the "20"

$$20\left[-\frac{t}{0.6}e^{-.6t} + \frac{1}{0.6} \cdot \left(-\frac{1}{0.6}\right)e^{-.6t}\right]_0^{10}$$

Joel

$$= 20\left[-\frac{10}{0.6}e^{-.6(10)} - \frac{1}{(.6)^2}e^{-.6(10)} + \frac{1}{(.6)^2}\right]$$

$$= 20\left[-\frac{10}{.6}e^{-6} - \frac{1}{.36}e^{-6} + \frac{1}{.36}\right]$$

9.4 #s 4, 7, 8, 13, 15, 18

≈ 54.6

≈ 54.59159638