3 hours

DAVID: You get to bring a one-page cheat sheet to the test.

$$\int \frac{x-5}{(x+2)^2(x-1)}\,dx$$

$$\frac{x-5}{(x+2)^2(x-1)} = \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{x-1} \Rightarrow$$

$$x-5 = A(x+2)(x-1) + B(x-1) + C(x+2)^2$$

Equating Coefficients

$$\Rightarrow x-5 = A(x^2+x-2) + Bx - B + C(x^2+4x+4)$$

$$\Rightarrow x-5 = Ax^2 + Ax - 2A + Bx - B + Cx^2 + 4Cx + 4C$$

$$\Rightarrow Ax^2 + Cx^2 = 0 \Rightarrow (A+C)x^2 = 0$$

$$\Rightarrow A + C = 0$$

$$\Rightarrow Ax + Bx + 4Cx = x$$

$$\Rightarrow (A+B+4C)\cancel{x} = \cancel{x}\ 1$$

$$\Rightarrow A+B+4C = 1$$

$$-2A - B + 4C = -5$$

$$A+C=0$$

$$A+B+4C=1$$

$$-2A-B+4C=-5$$

$$A+C=0 \Rightarrow C=-A$$

So

$$A+B+4C = A+B-4A = 1$$

$$-2A-B+4C = -2A-B-4A = -5$$

This gives

$$-3A+B=1 \Rightarrow B = 3A+1$$

$$-6A-B=-5$$

So $-6A-B = -6A-(3A+1) = -5$

$$\Rightarrow -9A-1=-5$$

$$-9A=-4$$

$$\boxed{A=\frac{4}{9}}$$

$$A = \frac{4}{9} \Rightarrow$$

$$B = 3\left(\frac{4}{9}\right)+1$$

$$= \frac{4}{3}+\frac{3}{3} = \boxed{\frac{7}{3} = B}$$

$$C = -A = \boxed{-\frac{4}{9} = C}$$

Now, $\displaystyle\int \frac{x-5}{(x+2)^2(x-1)}\,dx = \int\left(\frac{4}{9}\cdot\underbrace{\frac{1}{x+2}}_{\ln} + \frac{7}{3}\cdot\underbrace{\frac{1}{(x+2)^2}}_{u=(x+2)^2} - \frac{4}{9}\cdot\underbrace{\frac{1}{x-1}}_{\ln}\right)dx$

Good for Take-home part.

#s 39-50, § 8.4 we can do without

$$\int\sqrt{\frac{x+1}{x-1}}\,dx$$

$$\int e^x \cos x \, dx =$$

$u = e^x$, $du = e^x dx$

$dv = \cos x \, dx$, $v = \sin x$

$$uv - \int v\,du = e^x \sin x - \int e^x \sin x \, dx + C^*$$

$u = e^x$, $du = e^x dx$

$dv = \sin x \, dx$, $v = -\cos x$

$$= e^x \sin x - \left[uv - \int v\,du \right]$$

$$= e^x \sin x - \left[e^x(-\cos x) - \int (-\cos x)(e^x dx) \right] + C^*$$

$$= e^x \sin x + e^x \cos x - \int e^x \cos x \, dx + C^* \quad \text{Cewl!}$$

$$\int e^x \cos x \, dx = e^x \sin x + e^x \cos x - \int e^x \cos x \, dx + C^*$$

$$+ \int e^x \cos x \, dx \qquad = \qquad + \int e^x \cos x \, dx$$

$$2 \int e^x \cos x \, dx = e^x \sin x + e^x \cos x + C^*$$

$$\int e^x \cos x \, dx = \frac{1}{2}\left[e^x \sin x + e^x \cos x \right] + C, \quad \text{where } C = \frac{C^*}{2}$$

$$\int x \tan x \, dx \,^*$$

$u = x$, $dv = \tan x \, dx$

$du = dx$, $\ln|\sec x| = v$

$$\int \frac{\sin x}{\cos x} dx = -\int \frac{-\sin x \, dx}{\cos x}$$

$$= -\int \frac{du}{u} = -\ln|u| + C$$

$$= -\ln|\cos x| + C = \ln|\sec x| + C$$

$$\int x \arctan x \, dx \checkmark$$

$u = \arctan x$, $du = \frac{1}{1+x^2} dx$

$dv = x$, $v = \frac{1}{2}x^2$

So, $\displaystyle\int \frac{x-5}{(x+2)^2(x-1)}\,dx$ is good take-home material.

§8.7 is take-home stuff.

$\sin(2\theta) = 2\sin\theta\cos\theta$

Table of integrals #s 1–20

The Cheat Sheet from Test 1.

$\sin A\cos B = \frac{1}{2}[\sin(A-B)+\sin(A+B)]$

Trig Subst.

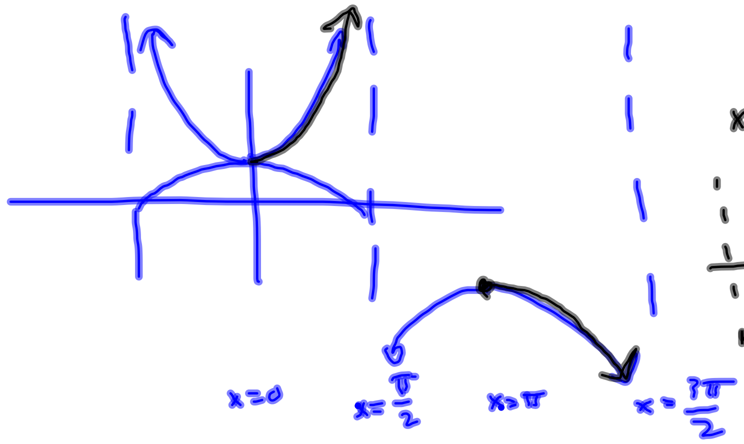
$\sin^2\theta+\cos^2\theta=1$

$\cos\theta = \sqrt{1-x^2} = \sqrt{\cos^2\theta}$ $x=\sin\theta$

$\tan\theta = \sqrt{x^2-1} = \sqrt{\tan^2\theta}$ $x=\sec\theta$

$\sec\theta = \sqrt{x^2+1} = \sqrt{\sec^2\theta}$ $x=\tan\theta$

$\rightarrow -\frac{\pi}{2} < \theta < \frac{\pi}{2}$



$x \in [0,\frac{\pi}{2}) \cup [\pi, \frac{3\pi}{2})$