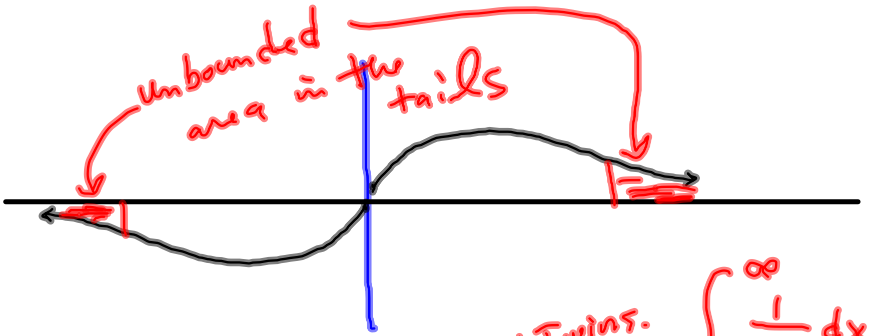
11. $\int_{-\infty}^{\infty} \frac{x}{1+x^2}\,dx$ is divergent! Even though, like David said, you'd expect it were zero,

$\int_{-b}^{b} \frac{x}{1+x^2}\,dx = 0$ for <u>any</u> fixed, real b.

But
$$\int_{-\infty}^{\infty} \frac{x}{1+x^2}\,dx = \int_{-\infty}^{0} \frac{x}{1+x^2}\,dx + \int_{0}^{\infty} \frac{x}{1+x^2}\,dx$$

But NEITHER of these converges!



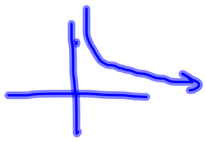
$\int_{1}^{\infty} \frac{1}{x}\,dx$ has same issue.

Evil Twins.

$\sum_{n=1}^{\infty} \frac{1}{n}$ diverges

But $\int_{1}^{\infty} \frac{1}{x^2}\,dx$ doesn't

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges



$\sum_{n=k}^{\infty} \frac{1}{n}$ is the "k-tail"

$\int_{k}^{\infty} \frac{1}{x}\,dx$

$$\int_{-\infty}^{\infty} x^3 e^{-x^4}\,dx =$$

Jonathan

$g(-x) = (-x)^3 = -x^3 = -g(x)$ ODD

$f(-x) = e^{-(-x)^4} = e^{-x^4} = f(x)$ is even

$$-\frac{1}{4}\int 4x^3 e^{-x^4}\,dx = -\frac{1}{4}\int e^u\,du = -\frac{1}{4}e^u + C$$

$$\int_{-\infty}^{\infty} \sim = \int_{-\infty}^{0} \sim + \int_{0}^{\infty} \sim$$

$$= \lim_{a\to-\infty}\int_a^0 x^3 e^{-x^4}\,dx + \lim_{b\to\infty}\int_0^b x^3 e^{-x^4}\,dx$$

$$= \lim_{a\to-\infty}\left[-\frac{1}{4}e^{-x^4}\right]_a^0 + \lim_{b\to\infty}\left[-\frac{1}{4}e^{-x^4}\right]_0^b$$

$$= \lim_{a\to-\infty}\left[-\frac{1}{4}\left[e^0 - e^{-a^4}\right]\right] + \lim_{b\to\infty}\left[-\frac{1}{4}\left(e^{-b^4} - e^0\right)\right]$$

$$= -\frac{1}{4}[1-0] + \left[-\frac{1}{4}(0-1)\right] = 0$$

$$e^{-(-1000)^4} = e^{-1000^4}$$

Why does the book say this diverges?

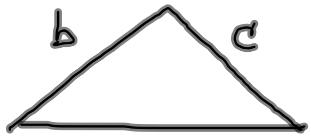
Look @ TYPE 1 Definition...

Teacher lied.

$f(x) = \cos(x^2)$. We need $\overbrace{\text{a}}^{\text{upper}}$ bound on $|f''(x)|$ in order to get a lower bound on n = the number of subintervals.

$$|E_T| \le \frac{K(b-a)^3}{12n^2} \le .0001 \quad \text{Need } K$$

$$|E_M| \le \frac{K(b-a)^3}{24n^2}$$



$f'(x) = -2x\sin(x^2) \Longrightarrow$

$|f''(x)| = |+2\sin(x^2) + 4x^2\cos(x^2)|$

$\le |2\sin(x^2)| + |4x^2\cos(x^2)|$

$\le 2 + 4|x^2|\,|\cos(x^2)|$

$\boxed{\le 2 + 4 = 6 \equiv K}$

$|a| \le |b| + |c|$

Triangle Inequality

$|a+b| \le |a| + |b|$

$|6| = |5-11| \le |5| + |11|$
$= 16$

Joel & I also looked @ $|f''(x)|$ on a calculator & found its bound was 3.8-ish.

18 $\displaystyle\int_0^\infty \frac{dz}{z^2+3z+2}$

$$\frac{1}{(z+1)(z+2)} = \frac{A}{z+1} + \frac{B}{z+2}$$

$$1 = A(z+2) + B(z+1)$$

$z=-2 \Longrightarrow$

$1 = -B \Longrightarrow B = -1$

$z = -1 \Longrightarrow$

$1 = A$

So we have $\displaystyle\int \frac{dz}{z+1} - \int \frac{dz}{z+2}$

$$= \ln|z+1| - \ln|z+2| + C$$

$$\therefore \int_0^\infty \frac{dz}{z^2+3z+2} = \lim_{t\to\infty} \Big[\ln(z+1) - \ln(z+2)\Big]_0^t$$

$$= \lim_{t\to\infty} \Big[\ln(t+1) - \ln(t+2) \underbrace{- (\ln 1 - \ln 2)}\Big]$$

$$= \lim_{t\to\infty} \Big[\ln(t+1) - \ln(t+2)\Big] + \ln 2$$

$$= \ln 2 + \lim_{t\to\infty}\left[\ln\left(\frac{t+1}{t+2}\right)\right] = \underbrace{\ln\left(\lim_{t\to\infty}\left(\frac{t+1}{t+2}\right)\right)}_{\ln 1 = 0} + \ln 2$$

$$= \ln 2$$

$$\lim_{t\to\infty}\left(\frac{t+1}{t+2}\right) = \lim_{t\to\infty}\left(\frac{\cancel{t}(1+\frac{1}{t})}{\cancel{t}(1+\frac{1}{t})}\right) = 1$$

$\frac{1}{t} \to 0$

$$\lim_{t\to\infty}\left(\frac{t+1}{t+2}\right) = 1$$