§8.8 #s 3, 5, 8, 11, 14, 18, 21, 25, 28, 36

Improper Integrals

$\boxed{\text{TYPE 1}}$ $\int_a^{\infty}$, $\int_{-\infty}^{b}$, $\int_{-\infty}^{\infty}$

$\boxed{\text{TYPE 2}}$ is where there's a discontinuity in the integrand, and not so much about the limits of integration being infinite

Last time we saw one of TYPE 2.

$$\int_2^3 \frac{dx}{\sqrt{3-x}} \qquad \text{Prob. @ } x=3,$$

Main theme is evaluate antiderivative & take a limit.

Convergent or Divergent?

$\cancel{\int_3^{\infty} \frac{1}{2x-5}\,dx}$

$f(x) = \frac{1}{2x-5} \leq \frac{1}{2x}$

$\&\ \frac{1}{2}\int_3^{\infty} \frac{1}{x}\,dx$ diverges

p-test. $p = 1$.

So $\int_3^{\infty} \frac{1}{2x-5}\,dx$ diverges

Does ↗ Converge? No.

Does

$$\int_1^{\infty} \frac{1}{x^2+1}\,dx$$

converge?

$\frac{1}{x^2+1} \leq \frac{1}{x^2}$ & $\int_1^{\infty} \frac{1}{x^2}\,dx$ converges, so $\boxed{\text{YES}}$

$\frac{1}{2x-5} \geq \frac{1}{2x}$

P-Test

$\int_a^{\infty} \frac{1}{x^p}\,dx$ converges if $p > 1$

otherwise, Diverges.

Comparison Test

If $f \geq g$ & $f, g \geq 0$

$\int_a^b f(x)\,dx$ converges

then $\int_a^b g(x)\,dx$ converges.

If $\int_a^b g(x)\,dx$ diverges,

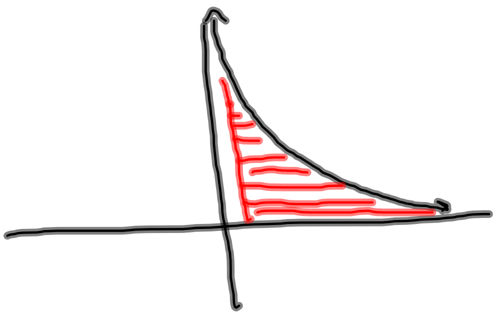
then $\int_a^b f(x)\,dx$ does, too

$$\int_1^{\infty} \frac{1}{x^2}\,dx = ?$$

$$= \lim_{b\to\infty} \int_1^b \frac{1}{x^2}\,dx = \lim_{b\to\infty} \left[-\frac{1}{x} \right]_1^b$$

$$= \lim_{b\to\infty} \left[\left(-\frac{1}{b}\right) \to 0 - \left(-\frac{1}{1}\right) \right]$$

$$= 0 + 1 = 1$$



17. $\int_1^\infty \frac{x+1}{x^2+2x}\,dx$

One approach:
Find antiderivative separately
& then take the limit.

$$\int \frac{x+1}{x^2+2x}\,dx = \frac{1}{2}\int \frac{2x+2}{x^2+2x}\,dx = \frac{1}{2}\int \frac{du}{u}$$

↳ Divergent by P-Test.

$$= \frac{1}{2}\ln|u| + C$$

$$= \frac{1}{2}\ln|x^2+2x| + C$$

So, $\frac{1}{2}\lim_{b\to\infty}\left[\ln|b^2+2b| - \ln|1^2+2(1)|\right]$

$= \infty$, i.e. $\int_1^\infty \frac{x+1}{x^2+2x}\,dx$ diverges.

11. $\int_{-\infty}^{\infty} \frac{x}{1+x^2}\,dx$ is divergent! Even though, like David said, you'd expect it were zero,

$$\int_{-b}^{b} \frac{x}{1+x^2}\,dx = 0$$ for $\underline{\text{any}}$ fixed, real b.

But

$$\int_{-\infty}^{\infty} \frac{x}{1+x^2}\,dx = \int_{-\infty}^{0} \frac{x}{1+x^2}\,dx + \int_{0}^{\infty} \frac{x}{1+x^2}\,dx$$

But NEITHER of these converges!

$$\int_{-\infty}^{\infty} x^3 e^{-x^4}\,dx =$$

Jonathan

$$-\frac{1}{4}\int 4x^3 e^{-x^4}\,dx = \frac{-1}{4}\int e^u\,du = \underline{\frac{-1}{4}e^u + C}$$

$$\int_{-\infty}^{\infty} = \int_{-\infty}^{0} + \int_{0}^{\infty}$$

$$= \lim_{a\to-\infty}\int_a^0 x^3 e^{-x^4}\,dx + \lim_{b\to\infty}\int_0^b x^3 e^{-x^4}\,dx$$

$$= \lim_{a\to-\infty}\left[-\frac{1}{4}e^{-x^4}\right]_a^0 + \lim_{b\to\infty}\left[-\frac{1}{4}e^{-x^4}\right]_0^b$$

$$= \lim_{a\to-\infty}\left[-\frac{1}{4}\left[e^0 - e^{-a^4}\right]\right] + \lim_{b\to\infty}\left[-\frac{1}{4}\left(e^{-b^4} - e^0\right)\right]$$

$$= -\frac{1}{4}[1 - 0] + \left[-\frac{1}{4}(0 - 1)\right] = 0$$

$$e^{-(-1000)^4} = e^{-1000^4}$$

Why does the book say this diverges?

Look @ TYPE I Definition...