Let $f(x) = \sin(4x)$. Find a lower bound on $n \ni$

(i) $|E_T| < .0001$

(ii) $|E_M| < .0001$

(iii) $|E_S| < .0001$

$$\int_0^{\frac{8\pi}{3}} \sin(4x)\,dx$$

$\sin(4x) = f(x) \Longrightarrow$

$f'(x) = 4\cos(4x) \Longrightarrow$

$f''(x) = -16\sin(4x) \Longrightarrow |f''(x)| \le |-16| = 16 = K$ for E_M & E_T

$f'''(x) = -64\cos(4x)$

$f^{(4)}(x) = 256\sin(4x) \Longrightarrow |f^{(4)}(x)| \le 256 = K$

(i) $|E_T| \le \dfrac{K(b-a)^3}{12n^2} = \dfrac{16\left(\frac{8\pi}{3}\right)^3}{12n^2}$ want $\le .0001$

$$\Longrightarrow \frac{16\left(\frac{8\pi}{3}\right)^3}{(12)(.0001)} \le n^2$$

$$\Longrightarrow n \ge \sqrt{\frac{16\left(\frac{8\pi}{3}\right)^3}{(12)(.0001)}} \approx 2799.930658$$

Let $n = 2800$ be lower bound.

$\boxed{E}$ Estimate $\int_0^{\pi} x^2 \sin x \, dx$, using $n=4$ intervals by the following methods:

(i) Trapezoid Rule
(ii) Midpoint Rule
(iii) Simpson's Rule

(i) want $T_4 = \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)\right]$

$\Delta x = \frac{b-a}{n} = \frac{\pi - 0}{4} = \frac{\pi}{4}$ $\to x_k = a + k\Delta x$, $x_k = x_{k-1} + \Delta x$

$x_0 = a = 0 \implies f(x_0) = 0^2 \cdot \sin 0 = 0 = f(x_0)$

$x_1 = a + \Delta x = 0 + \frac{\pi}{4} = \frac{\pi}{4}$

$f(x_1) = \left(\frac{\pi}{4}\right)^2 \sin\frac{\pi}{4} = \frac{\pi^2}{16}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}\pi^2}{32}$

$x_2 = x_1 + \frac{\pi}{4} = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

$f(x_2) = \left(\frac{\pi}{2}\right)^2 \sin\frac{\pi}{2} = \frac{\pi^2}{4}$

$x_3 = x_2 + \frac{\pi}{4} = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

$f(x_3) = \left(\frac{3\pi}{4}\right)^2 \sin\left(\frac{3\pi}{4}\right) = \frac{9\pi^2}{16}\cdot\frac{\sqrt{2}}{2}$

$x_4 = x_3 + \frac{\pi}{4} = \pi$

$f(x_4) = \pi^2 \sin(\pi) = 0$

$\therefore T_4 = \frac{\frac{\pi}{4}}{2}\left[0 + 2\frac{\sqrt{2}\pi^2}{2\cdot 16} + 2\cdot\frac{\pi^2}{4} + 2\cdot\frac{9\sqrt{2}\pi^2}{2\cdot 16} + 0\right]$

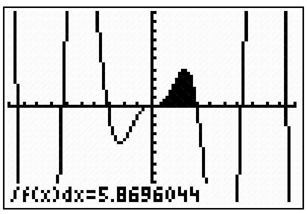
$= \frac{\pi}{8}\left[\frac{\sqrt{2}\pi^2}{16} + \frac{\pi^2}{2} + \frac{9\sqrt{2}\pi^2}{16}\right]$

$= \frac{\pi^3}{8\cdot 16}\left[\sqrt{2} + 8 + 9\sqrt{2}\right]$

$= \frac{\pi^3}{8\cdot 16}\left[10\sqrt{2} + 8\right] \approx 5.363634246 \approx T_4$

$\int_0^{\pi} x^2 \sin^2 x \, dx \approx 5.869604404$ by Maple

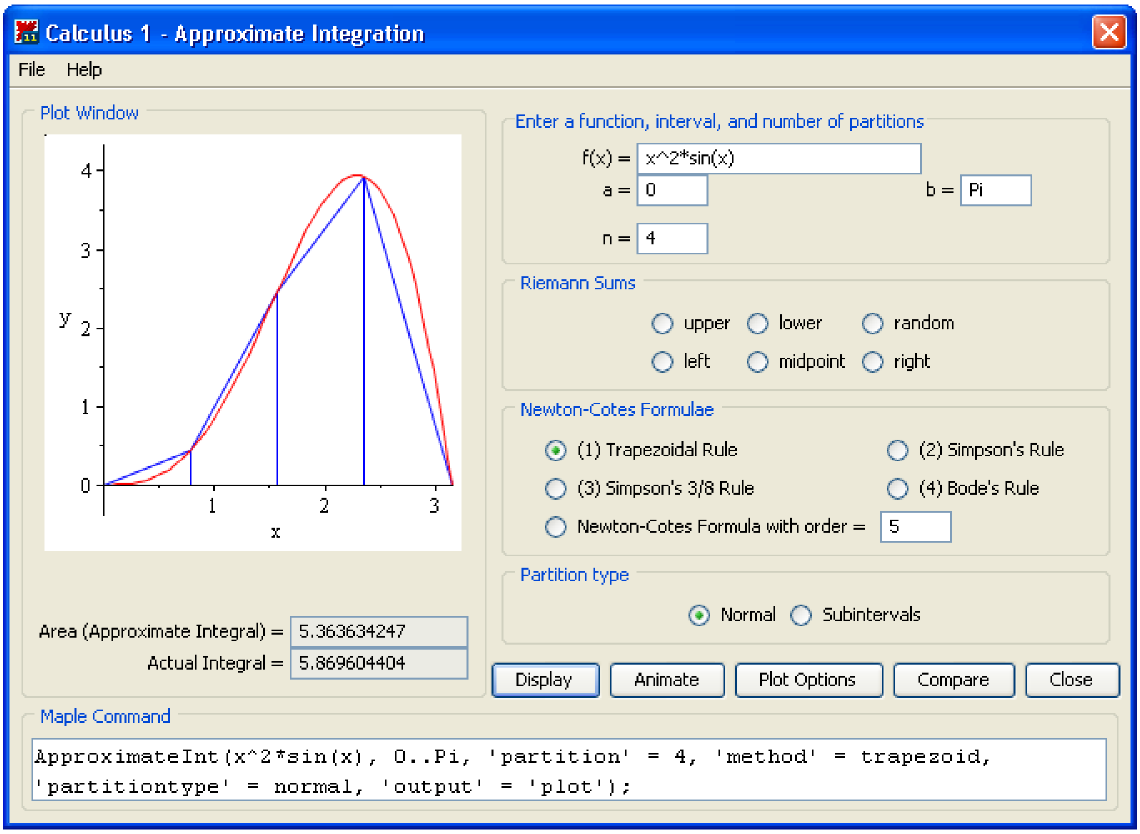
≈ 5.8696044 by TI-84



E_T = Actual − Estimate.

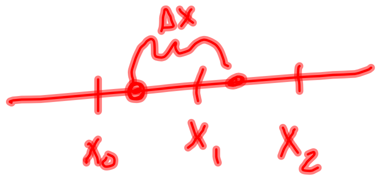
$\approx 5.8696044 - 5.3636342$

$= .5059702 \approx E_T$


Calculus 1 - Approximate Integration
File Help
Plot Window
Enter a function, interval, and number of partitions
f(x) = x^2*sin(x)
a = 0
b = Pi
n = 4
Riemann Sums
upper lower random
left midpoint right
Newton-Cotes Formulae
(1) Trapezoidal Rule
(2) Simpson's Rule
(3) Simpson's 3/8 Rule
(4) Bode's Rule
Newton-Cotes Formula with order = 5
Partition type
Normal Subintervals
Area (Approximate Integral) = 5.363634247
Actual Integral = 5.869604404
Display Animate Plot Options Compare Close
Maple Command
ApproximateInt(x^2*sin(x), 0..Pi, 'partition' = 4, 'method' = trapezoid,
'partitiontype' = normal, 'output' = 'plot');

(ii) Midpoint Rule

$\bar{x}_1 = a + \frac{\Delta x}{2} = 0 + \frac{\pi/4}{2} = \frac{\pi}{8}$



$f(x_1) \approx .0590145951$

$\bar{x}_2 = x_1 + \frac{\pi}{4} = \frac{3\pi}{8}$

```
      .0590145951
Y1(3π/8)
      1.282264523
Y1(5π/8)
      3.561845898
Y1(7π/8)
      2.891715161
```

$f(x_2) \approx 1.282264523$

$\bar{x}_3 = x_2 + \frac{\pi}{4} = \frac{5\pi}{8}$

$f(x_3) \approx 3.561845898$

$\bar{x}_4 = x_3 + \frac{\pi}{4} = \frac{7\pi}{8}$

$\sin^2\left(\frac{\pi}{8}\right)$

$f(x_4) \approx 2.891715161$

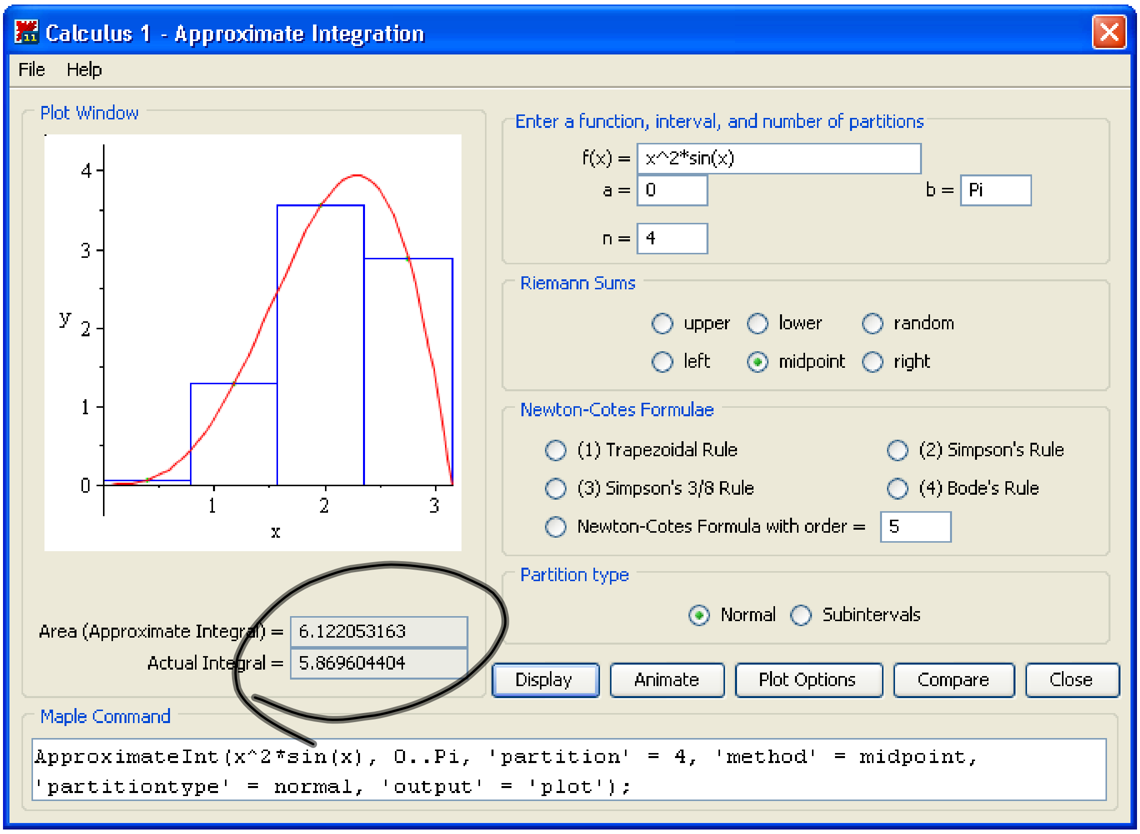
$= \frac{1}{2}\left[1 - \cos\frac{\pi}{4}\right]$

$M_4 = \Delta x \left[\sum_{k=1}^{4} f(\bar{x}_k)\right]$

$\approx \frac{\pi}{4}\left[7.794840177\right] \approx 6.122053159 \approx M_4$

$M_4 \approx 6.122053159$

$E_M = \int_0^{\pi} x^2 \sin x \, dx - M_4$


Calculus 1 - Approximate Integration
File Help
Plot Window
Enter a function, interval, and number of partitions
f(x) = x^2*sin(x)
a = 0
b = Pi
n = 4
Riemann Sums
upper lower random
left midpoint right
Newton-Cotes Formulae
(1) Trapezoidal Rule
(2) Simpson's Rule
(3) Simpson's 3/8 Rule
(4) Bode's Rule
Newton-Cotes Formula with order = 5
Partition type
Normal Subintervals
Area (Approximate Integral) = 6.122053163
Actual Integral = 5.869604404
Display Animate Plot Options Compare Close
Maple Command
ApproximateInt(x^2*sin(x), 0..Pi, 'partition' = 4, 'method' = midpoint,
'partitiontype' = normal, 'output' = 'plot');

(iii) Simpson's Rule

$$S_4 = \frac{\Delta x}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + y_4\right]$$

$$= \frac{\pi}{12}\left[0 + 4\cdot\frac{\pi^2}{16}\cdot\frac{\sqrt{2}}{2} + 2\cdot\frac{\pi^2}{4} + 4\cdot\frac{9\pi^2}{16}\cdot\frac{\sqrt{2}}{2} + 0\right]$$

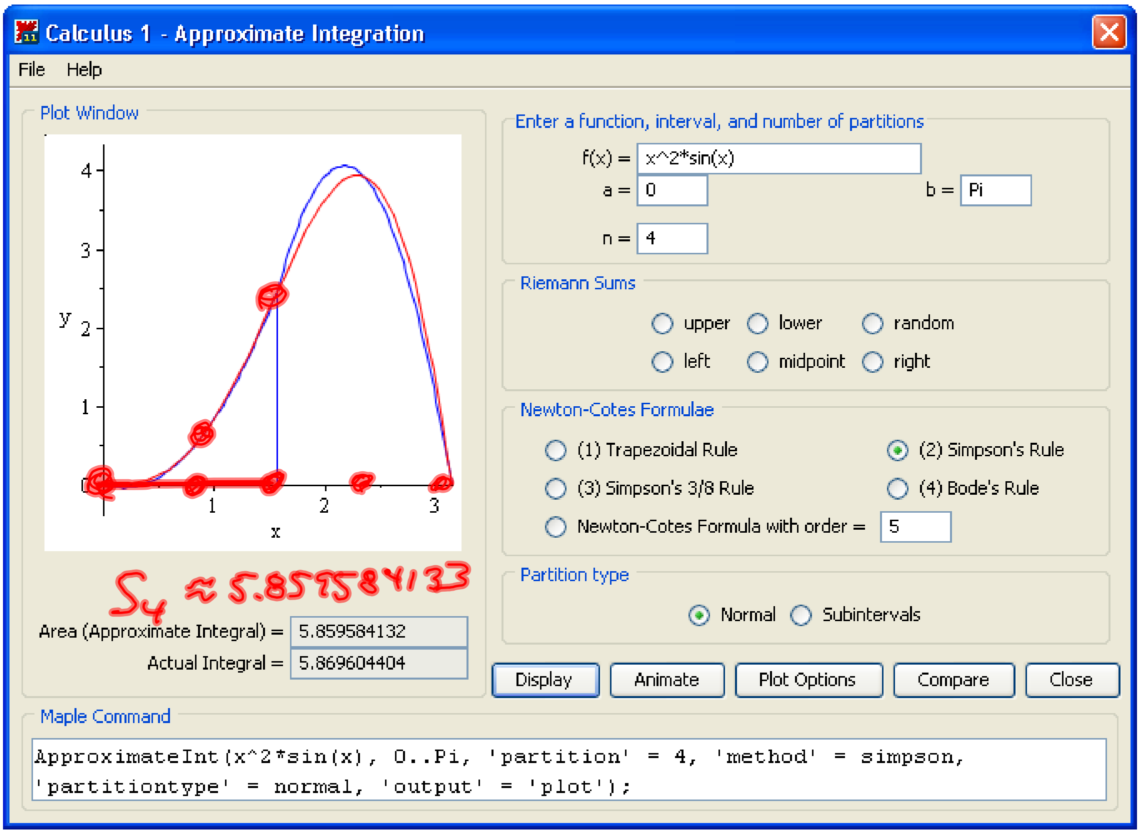
$$= \frac{\pi}{12}\left[\frac{\sqrt{2}\pi^2}{8} + \frac{\pi^2}{2} + \frac{9\sqrt{2}\pi^2}{8}\right]$$

$$= \frac{\pi}{12}\cdot\frac{\pi^2}{8}\left[\sqrt{2} + 4 + 9\sqrt{2}\right]$$

$$= \frac{\pi^3}{12\cdot 8}\left[10\sqrt{2} + 4\right] \approx 5.859584133$$

$S_4 \approx 5.859584133$

$E_S = \text{ACTUAL} - S_4$



§8.8 #s 3, 5, 8, 11, 14, 18, 21, 25, 28, 36

$$\int_2^3 \frac{dx}{\sqrt{3-x}} \qquad \left(\begin{array}{ll} u = 3-x & x=2 \to u=1 \\ du = -dx & x=3 \to u=0 \end{array} \right)$$

$$= \int_1^0 \frac{-du}{\sqrt{u}} = \int_0^1 u^{-\frac{1}{2}} du = \lim_{t \to 0^+} 2u^{\frac{1}{2}} \Big|_t^1$$

$$= 2 - \lim_{t \to 0^+} 2t^{\frac{1}{2}} = 2$$

$\to 0$