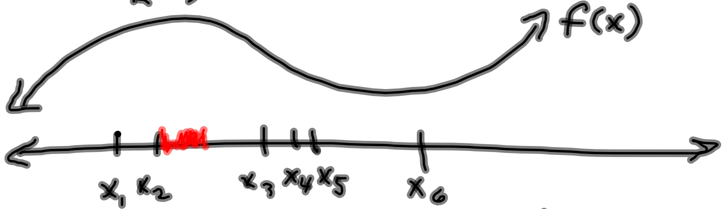
§8.7 Approximate Integration.

Recall, $f(x)$ cnt$\underline{s}$ on $[a,b]$, $f(x) \geq 0$. Then

Area between $f(x)$ & x-axis is approximately

$$\sum_{k=1}^{n} f(x_k^*)\Delta x_k \text{, where } x_k^* \in [x_{k-1}, x_k]$$ is ANY x in that interval.



If widths are the same, we have

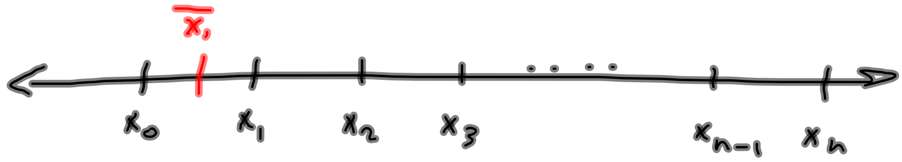
$$\text{Area} \approx \sum_{k=1}^{n} f(x_k^*)\Delta x = \sum_{k=1}^{n} f(x_k)\left(\frac{b-a}{n}\right)$$

$$= \boxed{\left(\frac{b-a}{n}\right)\sum_{k=1}^{n} f(x_k^*)} \xrightarrow{n \to \infty} \text{Area} = \int_a^b f(x)\,dx$$

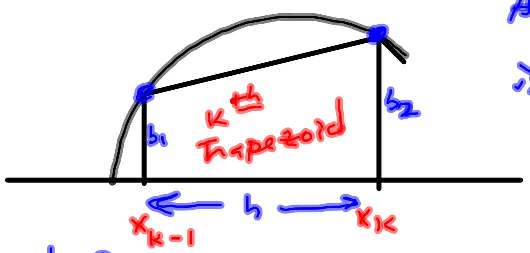
→ Today

Midpoint Rule

$$M_n = \Delta x \left[\sum_{k=1}^{n} f(\bar{x}_k) \right], \text{ where } \bar{x}_k = \frac{1}{2}\left[x_{k-1} + x_k\right]$$



Trapezoidal Rule



Area of the k^{th} trapezoid is $\frac{1}{2}(b_1 + b_2)h$

$$= \frac{1}{2}\left(f(x_{k-1}) + f(x_k)\right)\Delta x$$

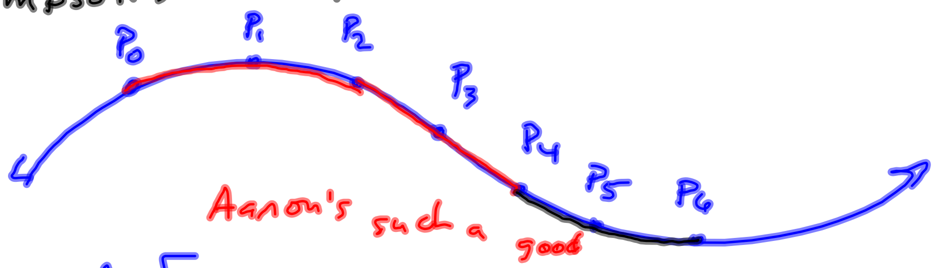
$$= A_k$$

$$\frac{b-a}{n} = \Delta x = h = x_k - x_{k-1}$$

$$\sum_{k=1}^{n} A_k = \frac{\Delta x}{2}\Big[f(x_0) + f(x_1) + f(x_1) + f(x_2) + f(x_2) + f(x_3) + f(x_3) + f(x_4) + \cdots + f(x_n)\Big]$$

$$= \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)\right]$$

Simpson's Rule

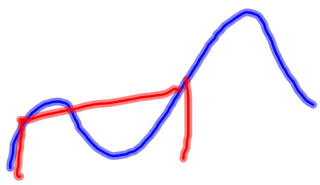


$\approx$

$$\text{Area} \approx \frac{\Delta x}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + 4y_5 + y_6\right]$$

$$\Delta x = \frac{b-a}{n}$$

Error Estimates



E_T = Error for Trapezoidal Rule

E_M = " " Midpoint Rule

E_S = " " Simpson's Rule

$$|E_T| \le \frac{K(b-a)^3}{12n^2}, \text{ where } K \ge |f''(x)| \ \forall x \in [a,b]$$

$$|E_M| \le \frac{K(b-a)^3}{24n^2} \quad " \quad " \quad " \quad " \quad "$$

$$|E_S| \le \frac{K(b-a)^5}{180n^4} \quad " \quad K \ge |f^{(4)}(x)| \ \forall x \in [a,b]$$

§8.7 #s 7, 12, 19, 20, 22, 30

#19 .9045242380 ? ?? $\pi^2 4 \approx$ 5.8696044
I think this is it.

#20 2.020058625

Let $f(x) = \sin(4x)$. Find a lower bound on $n \ni$

(i) $|E_T| < .0001$

(ii) $|E_M| < .0001$

(iii) $|E_S| < .0001$

$$\int_0^{\frac{8\pi}{3}} \sin(4x)\,dx$$

$\sin(4x) = f(x) \Longrightarrow$

$f'(x) = 4\cos(4x) \Longrightarrow$

$f''(x) = -16\sin(4x) \Longrightarrow |f''(x)| \leq |-16| = 16 = K$ for E_M & E_T

$f'''(x) = -64\cos(4x)$

$f^{(4)}(x) = 256\sin(4x) \Longrightarrow |f^{(4)}(x)| \leq 256 = K$

(i) $|E_T| \leq \dfrac{K(b-a)^3}{12n^2} = \dfrac{16\left(\frac{8\pi}{3}\right)^3}{12n^2}$ want $\leq .0001$

$$\Longrightarrow \frac{16\left(\frac{8\pi}{3}\right)^3}{(12)(.0001)} \leq n^2$$

$$\Longrightarrow n \geq \sqrt{\frac{16\left(\frac{8\pi}{3}\right)^3}{(12)(.0001)}}$$

Pick smallest integer that satisfies this inequality.