$$\int \frac{x}{x^4+x^2+1}\,dx = \frac{1}{2}\int \frac{du}{u^2+u+1}$$

$u = x^2 \rightsquigarrow du = 2x\,dx$

(Scratch: $u^2+u+1 = u^2+u+\left(\frac{1}{2}\right)^2 - \frac{1}{4} + 1$

$= \left(u+\frac{1}{2}\right)^2 + \frac{3}{4}$)

$$= \frac{1}{2}\int \frac{du}{\left(u+\frac{1}{2}\right)^2+\frac{3}{4}}$$

$t = u + \frac{1}{2}$

$dt = du$

$$= \frac{1}{2}\int \frac{dt}{t^2+\frac{3}{4}}$$

$\frac{3}{4} = \left(\frac{\sqrt{3}}{2}\right)^2$

(36) $\int \sin(4x)\cos(3x)\,dx$

$= \frac{1}{2}\int (\sin x + \sin(7x))\,dx$ etc.

(41) $\int \theta \tan^2\theta\, d\theta$ $\quad uv - \int v\,du$

$u = \theta \rightarrow du = d\theta$

$dv = \tan^2\theta\, d\theta$

$= (\sec^2\theta - 1)\,d\theta$

$v = \tan\theta - \theta$

$\theta(\tan\theta - \theta) - \int (\tan\theta - \theta)\,d\theta$

$= \theta\tan\theta - \theta^2 - \ln|\sec\theta| + \frac{1}{2}\theta^2 + C$

(46) $\int \frac{1+\sin x}{1-\sin x}\,dx$

$u = \sin x$

$du = \cos x\,dx$

$dx = \frac{du}{\cos x}$

$= \int \frac{1+u}{1-u}\cdot\frac{du}{\cos x}$

we love David.

$= \int \frac{1+u}{1-u}\cdot\frac{du}{\sqrt{1-u^2}}$

That looks hard

$\int \frac{1+\sin x}{1-\sin x}\cdot\frac{1+\sin x}{1+\sin x}\,dx = \int \frac{1+2\sin x+\sin^2 x}{\cos^2 x}\,dx$

$1-\sin^2 x = \cos^2 x$

$= \int \sec^2 x\,dx + 2\int \frac{\sin x}{\cos^2 x}\,dx + \int \frac{\sin^2 x}{\cos^2 x}\,dx + K$

$= A + B + C$

$A = \tan x + C_3$

$B = 2\int \frac{\sin x}{\cos x \cdot \cos x}\,dx = 2\int \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}\,dx$

$= 2\int \sec x \tan x\,dx = 2\sec x + C_1$

$C = \int \tan^2 x\,dx = \int (\sec^2 x - 1)\,dx = \tan x - x + C_7$

So, we have $A + B + C$

$= \tan x + 2\sec x + \tan x - x + K,$ where $K = C_3 + C_1 + C_7$

$\int \frac{\sin x}{\cos^2 x}\,dx$

$\int u^n\,du = \frac{1}{n+1}u^{n+1}$, except when $n = -1$

in which case, we have $\int u^{-1}\,du = \int \frac{du}{u} = \ln|u| + C$

$\int \frac{\sin x}{\cos^2 x}\,dx = -\int \frac{-\sin x\,dx}{\cos^2 x} = -\int \frac{du}{u^2}$

$= -\frac{u^{-1}}{-1} = \frac{1}{u} = \sec x$

$\int \sec x \tan x\,dx = \sec x$

$$\int \sin\sqrt{at}\,dt$$

$u = \sqrt{at}$

$du = \frac{1 \cdot a}{2\sqrt{at}}\,dt$

$dt = \frac{2\sqrt{at}}{a}\,du$

$=$

Let $u = at$ $du = a\,dt$

$\frac{1}{a}\int \sin\sqrt{u}\,du$

$s = \sqrt{u}$

$ds = \frac{1}{2\sqrt{u}}\,du$

$du = 2\sqrt{u}\,ds$

$= 2s\,ds$

$= \frac{1}{a}\int \sin s \; 2s\,ds$

$= \frac{2}{a}\int s \sin s\,ds$ is doable