0. Take a good look. Take a sip of your drink, and reflect on it. Might be something that jumps out at you, like even/odd function on an interval symmetric about the origin. You might see something that saves you a ton of work, later (Instructors *love* scary-looking test questions that are *easy*, if you can just *see* the lever...). $\int_{-2}^{2}(x^3 - 42.7\pi x^5 + 19x^{17})\,dx = 0$

1. Simplify the Integrand if Possible Sometimes the use of algebraic manipulation or trigonometric identities will simplify the integrand and make the method of integration obvious.

2. Look for an Obvious Substitution Try to find some function $u = g(x)$ in the integrand whose differential $du = g'(x)\,dx$ also occurs, apart from a constant factor.

3. Classify the Integrand According to Its Form If Steps 1 and 2 have not led to the solution, then we take a look at the form of the integrand $f(x)$.

a. Trig Functions – 8.2 skills.
b. Rational Functions – 8.4.
c. Integration by Parts – 8.1 – Look for powers of *x* times transcendental functions (, etc.)
d. Radicals – Trig substitution tricks (8.3) for situations and the "simplifying substitution" we spoke about briefly in 8.4: Use when facing situations that aren't cured by other techniques.

(21) $\int \arctan\sqrt{x}\,dx = \int (\arctan y)(2y\,dy)$

Let $y = \sqrt{x} \Rightarrow dy = \frac{1}{2\sqrt{x}}dx = \frac{1}{2y}dx$

$\Rightarrow dx = 2y\,dy$

$= 2\int y \arctan y \, dy$

1st instinct: Integrate by parts with $u = y$

$u = y \Rightarrow du = dy$

$dv = \arctan u \, du$

$\Rightarrow v = -\frac{1}{2}\ln(1+u^2)$

So, $uv - \int v\,du =$ OWIE!

$\int v\,du = -\frac{1}{2}\int \ln\underbrace{(1+u^2)}_{\text{Need } 2u\,du \text{ \& don't have it.}}du$

$\int \ln u \, du$

$= u\ln u - u + C$

$\int \ln x \, dx$

$= x\ln x - x + C$

Could continue play with

$\int \ln(1+x^2)dx$

$u = \ln(1+x^2)$ & $du = \frac{2x}{1+x^2}dx$

$dv = dx \longrightarrow v = x$

$uv - \int v\,du = x\ln(1+x^2) - \underbrace{\int x \cdot \frac{2x}{1+x^2}dx}_{\text{still kinda icky.}}$

Sissy!

You throw like a girl!

$$\int y \arctan y \, dy$$

Let $u = \arctan y \Rightarrow du = \frac{1}{1+y^2} dy$

$dv = y\,dy \Rightarrow v = \frac{1}{2}y^2$

$$\int u\,dv = uv - \int v\,du$$

$$= \frac{1}{2}y^2 \arctan y - \frac{1}{2}\int y^2 \cdot \frac{1}{1+y^2} dy$$

Dang! Same mess.

Let's look at $\int \frac{y^2}{1+y^2} dy = \frac{1}{2}\int y \cdot \frac{2y}{1+y^2} dy$

$u = y \rightsquigarrow du = dy$

$dv = \frac{2y}{1+y^2} dy \Rightarrow v = \ln(1+y^2)$

$uv - \int v\,du = y\ln(1+y^2) - \int \ln(1+y^2)\,dy$

$$\int \frac{y^2}{1+y^2} dy = \int \frac{-1+1+y^2}{1+y^2} dy = -\int \frac{1}{1+y^2} dy + \int \frac{1+y^2}{1+y^2} dy$$

$$= -\arctan y + y + C$$

Should've seen this on previous page

Still need to backtrack to "x"

26 $\displaystyle\int \frac{3x^2-2}{x^3-2x-8}\,dx$

$\displaystyle\int \frac{du}{u} = \ln|u|$

$= \ln|x^3-2x-8| + C$

$\displaystyle\int \frac{g'(x)}{g(x)}\,dx$

$x^3+1 = (x+1)(x^2-x+1)$

partial fractions

25 $\displaystyle\int \frac{3x^2-2}{x^2-2x-8}\,dx$

$= \displaystyle\int \frac{3x^2-2}{(x-4)(x+2)}\,dx$

$$\begin{array}{r|l} & 3 \quad r\ 6x+22 \\ x^2-2x-8 & 3x^2-0x-2 \\ & -(3x^2-6x-24) \\ \hline & 6x+22 \end{array}$$

$= \displaystyle\int \left(3 + \frac{6x+22}{x^2-2x-8}\right) dx$

partial fractions this

I forgot to divide. This told me

$$\frac{3x^2-2}{(x-4)(x+2)} = \frac{A}{x-4} + \frac{B}{x+2}$$

$3x^2-2 = A(x+2) + B(x-4)$

Can't be done. (Not if A & B are constant. No "x^2" & I need one for him

(31) $\displaystyle\int \sqrt{\frac{1+x}{1-x}}\,dx$

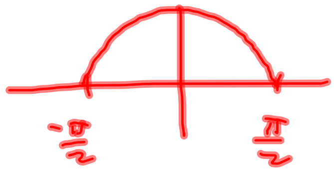
$u = \sqrt{\dfrac{1+x}{1-x}}$. makes a (messy) rational function out of it.

"Trick" Scratch

$$\frac{\sqrt{1+x}}{\sqrt{1-x}} \cdot \frac{\sqrt{1+x}}{\sqrt{1+x}} = \frac{1+x}{\sqrt{1-x^2}}$$

$x = \sin\theta \quad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$dx = \cos\theta \, d\theta$

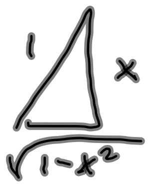


$$= \int \frac{(1+\sin\theta)(\cos\theta\, d\theta)}{\cos\theta}$$

$$= \int (1+\sin\theta)\, d\theta$$

$$= \theta - \cos\theta + C$$

$$= \arcsin x - \sqrt{1-x^2} + C$$



31

$$\boxed{u=\sqrt{\frac{1+x}{1-x}}}=\frac{\sqrt{1+x}}{\sqrt{1-x}} \qquad \int\sqrt{\frac{1+x}{1-x}}\,dx$$

$$\ln u = \frac{1}{2}\ln(1+x)-\frac{1}{2}\ln(1-x)$$

$$\frac{u'}{u}=\frac{1}{2}\cdot\frac{1}{1+x}+\frac{1}{2}\,\frac{1}{1-x}$$

$$du = \frac{1}{2}\left(\frac{1}{1+x}+\frac{1}{1-x}\right)\sqrt{\frac{1+x}{1-x}}\cdot dx$$

$$=$$

painful, but supposedly will work.

$f(x) = \sqrt{x}$

$$\frac{f(x+h) - f(x)}{h} = \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \frac{x + h - x}{h(\sqrt{x+h} + \sqrt{x})} = \frac{1}{\sqrt{x+h} + \sqrt{x}} \xrightarrow{h \to 0} \frac{1}{2\sqrt{x}}$$

$$= f'(x).$$