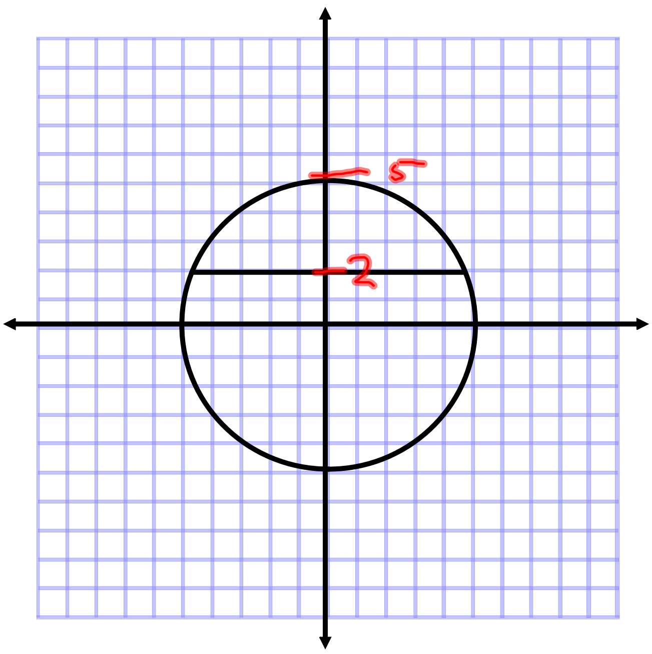
$$\frac{1}{2(12.9\pi)} \cdot \int_{-12.9\pi}^{12.9\pi} \sin^5 x \cos^{13} x \, dx = 0$$

It's odd. 0

8.3 Questions? #42

8.4 #5
4, 9, 20, 21, 27, 34, 40,
45*, 46, 48
$\boxed{\text{BONUS}}$ $\boxed{57-60}$



21 $\deg(\text{num}) > \deg(\text{denom})$: Divide

$\boxed{x-c}$

$$\int \frac{x^3+4}{x^2+4}\,dx$$

$$\left(\begin{array}{r|l} & x \qquad r\ -4x+4 \\ x^2+4 & x^3+0x^2+0x+4 \\ & -(x^3 \qquad\ +4x) \\ \hline & \qquad -4x+4 \end{array} \right)$$

$$= \int\left(x + \frac{-4x+4}{x^2+4}\right)dx = \int x\,dx - 4\int \frac{x-1}{x^2+4}\,dx$$

$$= \tfrac{1}{2}x^2 - \frac{4}{2}\int \frac{2x}{x^2+4}\,dx + 4\int \frac{dx}{x^2+4} \rightsquigarrow \tan\theta = \frac{x}{2}$$

$u = x^2+4$
$du = 2x\,dx$

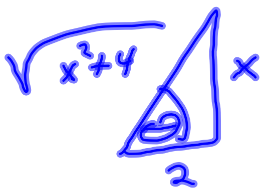
$x = 2\tan\theta$
$dx = 2\sec^2\theta\,d\theta$

$$= \boxed{\tfrac{1}{2}x^2 + 2\ln(x^2+4) + 2\tan^{-1}\left(\tfrac{x}{2}\right) + C} \quad x \in (-\pi, \pi)\ !?$$

Scratch

$$4\int \frac{dx}{x^2+4} = 4\int \frac{2\sec^2\theta\,d\theta}{2^2\tan^2\theta + 2^2} = 4\int \frac{2\sec^2\theta\,d\theta}{2^2(\tan^2\theta+1)}$$

$$= \frac{4}{4}\int \frac{2\sec^2\theta\,d\theta}{\sec^2\theta} = 2\int d\theta = 2\theta + C = 2\tan^{-1}\left(\tfrac{x}{2}\right) + C$$



39 $\displaystyle\int \frac{dx}{x\sqrt{x+1}}$

$u = \sqrt{x+1}$

$du = \frac{1}{2}\cdot\frac{dx}{\sqrt{x+1}} = \frac{dx}{2\sqrt{x+1}}$

$dx = 2\sqrt{x+1}\,du = 2u\,du$

Footwork

$u = \sqrt{x+1}$

$\Rightarrow u^2 = x+1$

$\Rightarrow u^2 - 1 = x$

$= \displaystyle\int \frac{2\cancel{u}\,du}{(u^2-1)\cancel{u}} = 2\int \frac{du}{u^2-1} =$

Clear Frac's.

Scratch $\frac{1}{u^2-1} = \left(\frac{1}{(u-1)(u+1)} = \frac{A}{u-1} + \frac{B}{u+1}\right)(u-1)(u+1)$

$\Rightarrow 1 = A(u+1) + B(u-1)$

Pick:

$u=1 \Rightarrow 1 = 2A + 0 \Rightarrow A = \frac{1}{2}$

$u=-1 \Rightarrow 1 = 0 + -2B \Rightarrow B = -\frac{1}{2}$

$= \cancel{2}\left[\cancel{\tfrac{1}{2}}\displaystyle\int \frac{du}{u-1} - \cancel{\tfrac{1}{2}}\int \frac{du}{u+1}\right] = \boxed{\ln|u-1| + \ln|u+1| + C}$

$= \ln\left|\sqrt{x+1} - 1\right| + \ln\left|\sqrt{x+1} + 1\right| + C$

$= \ln\sqrt{x} + \ln\sqrt{x} + 2$

$\sqrt{x^2+4} = x+2$

No!!!

47 Another cool substitution

$$\int \frac{e^{2x}}{e^{2x}+3e^x+2}\,dx$$

$e^{2x} = e^{x\cdot 2} = (e^x)^2$

$a^{bc} = (a^b)^c$

(Let $u = e^x \Rightarrow du = e^x dx$
$\Rightarrow dx = \frac{du}{e^x} = \frac{du}{u}$)

$$= \int \frac{u^2}{u^2+3u+2}\cdot\frac{du}{u}$$

$$= \int \frac{u\,du}{(u+1)(u+2)}$$

Factoring suck? Use Quadratic Formula & the Factor Theorem.

$u^2+3u+2=0$

$a=1, b=3, c=2 \Rightarrow$

$b^2-4ac = 3^2-4(1)(2)$
$= 9-8 = 1$

$x = \frac{-b \pm\sqrt{b^2-4ac}}{2a}$

$= \frac{-3\pm\sqrt{1}}{2(1)} = \frac{-3\pm 1}{2}$

$\frac{-3+1}{2} = -1$ $\frac{-3-1}{2} = -2$

$u=-1$ OR $u=-2$

So, u^2+3u+2
$= (u+1)(u+2)$

$u=-1$ is a zero $\Longleftrightarrow$ $(u+1)$ is a factor.

scratch

$\frac{u}{(u+1)(u+2)} = \frac{A}{u+1} + \frac{B}{u+2}$

$u = A(u+2) + B(u+1)$

$u=-2$:
$-2 = -B \Rightarrow B=2$

$u=-1$:
$-1 = A$

$$= \int\frac{2\,du}{u+2} + \int\frac{-1\,du}{u+1}$$

$$= \boxed{2\ln|u+2| - \ln|u+1| + C}$$

$= 2\ln|e^x+2| - \ln|e^x+1| + C$

$= 2\ln(e^x+2) - \ln(e^x+1) + C$

$\frac{2}{u+2} - \frac{1}{u+1}$

$= \frac{2(u+1) - 1(u+2)}{(u+2)(u+1)}$

$= \frac{2u+2-u-2}{(u+2)(u+1)} = \frac{u}{(u+2)(u+1)}$

$$\frac{u}{(u+2)(u+1)} = \frac{A}{u+2} + \frac{B}{u+1} \Longrightarrow$$

$u = A(u+1) + B(u+2)$

$u = Au + A + Bu + 2B \Longrightarrow$

$Au + Bu = u \Longrightarrow (A+B)u = u \Longrightarrow A+B = 1$

$A+2B = 0$

&

$A+2B = 0$

$$\begin{bmatrix} 1 & 1 & | & 1 \\ 1 & 2 & | & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & | & 1 \\ 0 & 1 & | & -1 \end{bmatrix}$$

$$\sim \begin{array}{cc} A & B \\ \end{array} \begin{bmatrix} 1 & 0 & | & 2 \\ 0 & 1 & | & -1 \end{bmatrix} \begin{array}{l} A=2 \\ B=-1 \end{array}$$

$A + 2B = 0$

$A = -2B$

$A + B = 1$

$-2B + B = 1$

$-B = 1$

$\boxed{B = -1}$

$A = -2(-1) = 2$

$A = 2$