$$\frac{1}{x+1} + \frac{3}{x-1} = \frac{x-1+3x+3}{(x+1)(x-1)} = \frac{4x+2}{(x+1)(x-1)}$$

START HERE

$$\int \frac{3}{x-1}\,dx = 3\ln|x-1| + C$$

$$\int \frac{dx}{x+1} = \ln|x+1| + C$$

$$\int \frac{4x+2}{x^2-1}\,dx = \ln|x+1| + 3\ln|x-1| + C$$

Need to capture this

$$\frac{x^4+1}{x^5+4x^3}$$

WANT $\frac{A}{x-b} + \frac{B}{x-c} + \frac{C}{x-d}$ is ideal

$x^5 + 4x^3 = x^3(x^2+4)$

$$\frac{x^4+1}{x^3(x^2+4)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{Dx+E}{x^2+4}$$

$$\frac{x+5}{(x-2)^2(x+3)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x+3}$$

If degree of numerator is greater, do some division first.

↳ or equal.

$$\int \frac{(x^2-3x+1)}{x+2}\,dx = \int \left(x-5+\frac{16}{x+2}\right)dx$$

$$\begin{array}{r|rrr} -2 & 1 & -3 & 1 \\ & & -2 & 10 \\ \hline & 1 & -5 & 16 \end{array}$$

(7) $\int \frac{x}{x-6}\,dx = \int \left(1 + \frac{6}{x-6}\right) dx$

$= x + 6\ln|x-6| + C$

$= \frac{x^2}{2} + 6\ln|x-6| + C$

6 | 1 0
6
1 6

(9) $\int \frac{x-9}{(x+5)(x-2)}\,dx =$

$\frac{x-9}{(x+5)(x-2)} = \frac{A}{x+5} + \frac{B}{x-2}$ $\quad (x+5)(x-2)$

$x-9 = A(x-2) + B(x+5) = Ax - 2A + Bx + 5B$

$= (A+B)x - 2A + 5B$

Equate coefficients

x: $A+B = 1$

-9: $-2A+5B = -9$

$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ -2 & 5 & -9 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 7 & -7 \end{array}\right]$

$\sim \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 1 & -1 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -1 \end{array}\right]$

$A = 2$

$B = -1$

Sick way

$A(x-2) + B(x+5) = x - 9$

$x=2$: $7B = -7 \Rightarrow B = -1$

$x=-5$: $-7A = -14$

$A = 2$

$A = 2$

$B = -1$

$A + B = 1$

$A = 1 - B$

$-2A + 5B = -9 \Rightarrow$

$-2(1-B) + 5B = -9$

$-2 + 2B + 5B = -9$

$7B = -7$

$B = -1 \Rightarrow$

$A = 1 - B$

$= 1 - (-1) = 2$

$A = 2$