$$= \frac{(4)(5)(3)}{2}\left[\int_0^{\frac{\pi}{2}} d\theta + \frac{1}{2}\int_0^{\frac{\pi}{2}} \cos(2\theta)\, 2d\theta\right]$$

$$= (2)(5)(3)\left[\theta\Big]_0^{\frac{\pi}{2}} + \frac{1}{2}\sin(2\theta)\right]_0^{\frac{\pi}{2}}$$

want the expression in x.

$$= (2)(5)(3)\left\{ \sin^{-1}\frac{x}{3}\Big]_0^3 + \frac{1}{2}\sin(2\theta)\Big]_0^{\frac{\pi}{2}} \right\}$$

$x = 3\sin\theta$

$\theta = \sin^{-1}\left(\frac{x}{3}\right)$

$x = 3\sin\theta$,

$$= 30\left\{ \sin^{-1}\frac{x}{3}\Big|_0^3 + \frac{1}{2}(2)\,x\sqrt{1-x^2}\Big|_0^3 \right\}$$

so $\sin(2\theta) = ?$

hold this thought!

$\sin(2\theta) = 2\sin\theta\cos\theta$

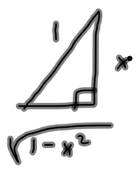
$= 2x$

$$= 30\left\{ \sin^{-1}(1) - \sin^{-1}(0) + 3\sqrt{1-3^2} \right. \;!?$$

That doesn't work, Steve.

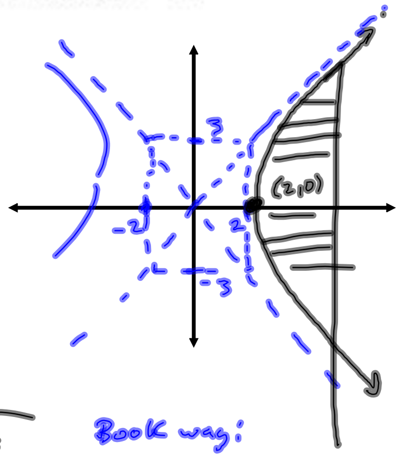
$\sin\theta = x = \frac{x}{1}$

$\cos\theta = \sqrt{1-x^2}$



34. Find the area of the region bounded by the hyperbola $9x^2 - 4y^2 = 36$ and the line $x = 3$.

$$\frac{x^2}{2^2} - \frac{y^2}{3^2} = 1$$



$$9x^2 - 4y^2 = 36$$
$$-4y^2 = 36 - 9x^2$$
$$y^2 = \frac{9}{4}x^2 - 9$$
$$y = \pm\sqrt{\frac{9}{4}x^2 - 9}$$
$$= \pm \frac{3}{2}\sqrt{x^2 - 2^2}$$

Book way: Let $u = \frac{3}{2}x$, giving $\sqrt{u^2-9}$

$$y = \frac{3}{2}\sqrt{x^2 - 2^2} \qquad x = 2\sec\theta$$

$$dx = 2\sec\theta\tan\theta\,d\theta$$

$$\frac{3}{2}\int_2^3 \sqrt{x^2 - 2^2}\,dx$$

$x = 2 \Rightarrow 2\sec\theta = 2$, $\sec\theta = 1$

$x = 3 \Rightarrow 2\sec\theta = 3$, $\sec\theta = \frac{3}{2}$

$$= \frac{3}{\cancel{2}}\int_0^{\sec^{-1}(\frac{3}{2})} \sqrt{2^2\sec^2\theta - 2^2}\cdot \cancel{2}\sec\theta\tan\theta\,d\theta$$

$$= 3\int_0^{\sec^{-1}(\frac{3}{2})} 2\tan\theta\sec\theta\tan\theta\,d\theta \; *$$

$$= 6\int_0^{\sec^{-1}(\frac{3}{2})} (\sec^2\theta - 1)\sec\theta\,d\theta \; *$$

$$\sec^{-1}\left(\tfrac{3}{2}\right) = \alpha$$

$$= 6\int_0^{\alpha} (\sec^3\theta - \sec\theta)\,d\theta$$

$$= 6\left[\frac{1}{2}(\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|) - \ln|\sec\theta + \tan\theta|\right]_0^{\alpha}$$

Should this be a 12?

$$\int \sec^2\theta\sec\theta\,d\theta$$

$u = \sec\theta \longrightarrow du = \sec\theta\tan\theta\,d\theta$

$dv = \sec^2\theta\,d\theta \longrightarrow v = \tan\theta$

Wow! What a big problem!

$$\int_0^{\frac{3\sqrt{3}}{2}} \frac{x^3\,dx}{(4x^2+9)^{3/2}}$$

Book:

$u = 2x$

$$\left\{ \begin{array}{l} \frac{1}{2}u = x \longrightarrow dx = \frac{1}{2}du \\ \frac{1}{8}u^3 = x^3 \end{array} \right\}$$

$$\int \frac{x^3\,dx}{(4x^2+9)^{3/2}} = \int \frac{\frac{1}{8}u^3 \cdot \frac{1}{2}\,du}{\left(\sqrt{u^2+9}\right)^3}$$

My way:

$$\left(2\sqrt{x^2+\frac{9}{4}}\right)^3$$