$$\int_0^{\frac{\pi}{3}} \tan^5 x \sec^4 x \, dx$$

$$= \int_0^{\frac{\pi}{3}} (\tan^2 x)^2 \tan x \sec^4 x \, dx$$

$$= \int_0^{\frac{\pi}{3}} (\sec^2 x - 1)^2 \tan x \sec^4 x \, dx$$

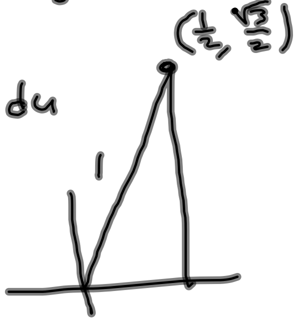
$$= \int_0^{\frac{\pi}{3}} (\sec^4 x - 2\sec^2 x + 1) \sec^3 x \underbrace{\sec x \tan x \, dx}_{du}$$

$$= \int_0^{\frac{\pi}{3}} \sec^7 x \sec x \tan x \, dx - 2\int_0^{\frac{\pi}{3}} \sec^5 x \sec x \tan x \, dx$$

$$+ \int_0^{\frac{\pi}{3}} \sec^3 x \sec x \tan x \, dx$$

$u = \sec x$, $x = 0 \Rightarrow u = \sec(0) = 1$, $x = \frac{\pi}{3} \Rightarrow \sec x = 2$

$$= \int_1^2 u^7 du - 2\int_1^2 u^5 du + \int_1^2 u^3 du$$



$$= \left[\frac{1}{8}u^8 - 2\cdot\frac{1}{6}u^6 + \frac{1}{4}u^4\right]_1^2$$

$$= \frac{1}{8}[256 - 1] - \frac{1}{3}[64 - 1] + \frac{1}{4}[16 - 1]$$

$(21)(8)$

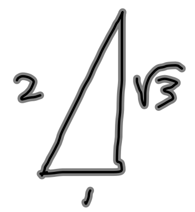
$$= \frac{1}{8}[255] - \frac{1}{3}[63] + \frac{1}{4}[15]$$

$$= \frac{255}{8} - \frac{168}{8} + \frac{30}{8} = \frac{285 - 168}{8} = \boxed{\frac{117}{8}}$$

$$\int_0^{\frac{\pi}{3}} \tan^5 x \sec^4 x \, dx$$

$$= \int_0^{\frac{\pi}{3}} \tan^5 x \sec^2 x \sec^2 x \, dx$$

$$= \int_0^{\frac{\pi}{3}} \tan^5 x \left(\tan^2 x + 1\right) \sec^2 x \, dx$$



$$= \int_0^{\frac{\pi}{3}} \Big[\left(\tan^7 x + \tan^5 x\right) \sec^2 x \, dx$$

$$= \left[\frac{\tan^8 x}{8} + \frac{\tan^6 x}{6}\right]_0^{\frac{\pi}{3}} = \frac{(\sqrt{3})^8}{8} + \frac{(\sqrt{3})^6}{6} - (0+0)$$

$$= \frac{81}{8} + \frac{27}{6} = \frac{81}{8} + \frac{9}{2} = \frac{81+36}{8} = \frac{117}{8}$$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^3 x\,dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot x\,(\csc^2 x - 1)\,dx$$

$-\ln|\sec x|$

$\ln|\cos x|$

$$= -\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot x\,(\csc^2 x\,dx) - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot x\,dx$$

$$= -\frac{1}{2}\cot^2 x\Big]_{\frac{\pi}{4}}^{\frac{\pi}{2}} - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x}\,dx$$

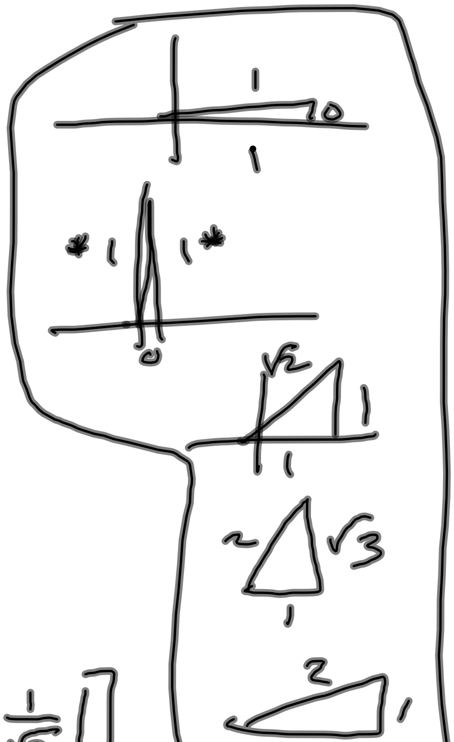
$$= -\frac{1}{2}\left[\cot^2\left(\frac{\pi}{2}\right) - \cot^2\left(\frac{\pi}{4}\right)\right]$$

$$-\Big[\ln|\sin x|\Big]_{\frac{\pi}{4}}^{\frac{\pi}{2}} =$$

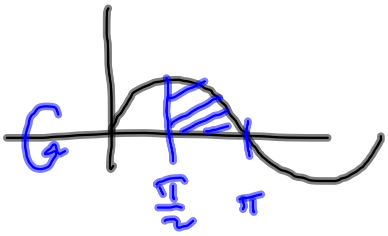
$$= -\frac{1}{2}\left[0^2 - 1^2\right] - \left[\ln|1| - \ln\left|\frac{1}{\sqrt{2}}\right|\right]$$

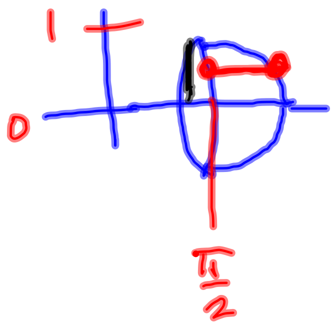
$$= \frac{1}{2} + \ln\left|\frac{1}{\sqrt{2}}\right|$$

$$= \frac{1}{2} - \ln|\sqrt{2}| = \frac{1}{2} - \ln\sqrt{2}$$



$y=\sin x$, $y=0$, $\frac{\pi}{2}\le x\le\pi$ about x-axis





Shell method:

$$2\int_0^1 \pi y(\quad)\,dy$$

$$=2\pi\int_0^1 y\left(\sin^{-1}y-\frac{\pi}{2}\right)dy$$

Disc Method

$$\pi\int_{\frac{\pi}{2}}^{\pi}(\sin x)^2\,dx$$

$$=\pi\int_{\frac{\pi}{2}}^{\pi}\left(\frac{1}{2}(1-\cos(2x))\right)dx$$

$$\int y\arcsin y\,dy$$

$$\frac{d}{dy}\sin^{-1}y=\frac{1}{\sqrt{1-y^2}}$$

$$\int y\sin^{-1}y\,dy=uv-\int v\,du=\frac{1}{2}y^2\sin^{-1}y-\frac{1}{2}\int y^2\frac{1}{\sqrt{1-y^2}}\,dy$$

$u=\sin^{-1}y$ $\quad dv=y\,dy$

$du=\frac{1}{\sqrt{1-y^2}}dy$ $\quad v=\frac{1}{2}y^2$

No! x

$$=\frac{\pi}{2}\int_{\frac{\pi}{2}}^{\pi}(1-\cos(2x))\,dx=\frac{\pi}{2}\left[\frac{x^2}{2}-\frac{1}{2}\sin(2x)\right]_{\frac{\pi}{2}}^{\pi}$$

$$=\frac{\pi}{2}\left[\pi-0-\left(\frac{\pi}{2}-0\right)\right]=\frac{\pi}{2}\left[\frac{\pi}{2}\right]=\frac{\pi^2}{4}$$

$\frac{1}{2}\sin(2\pi)$ $\quad$ $\frac{1}{2}\left(\sin\left(2\frac{\pi}{2}\right)\right)$