8.2 II TRIGONOMETRIC INTEGRALS

we've done

$\int \sin^n x \cos x\, dx$, $\int \cos^n x \sin x\, dx$

$\int \sin^{2m+1} x \cos^{2n+1} x\, dx$

$\int \sin^3 x \cos^3 x\, dx$

$(1-\sin^2 x)\cos x$

Then use $\int \sin^m x \cos x dx$

$= \int u^m du$

$= \frac{\sin^{m+1} x}{m+1} + C$

$\int \sin^{2m+1} x\, dx$

$= \int \underbrace{\sin^{2m} x}_{\text{convert to } (1-\cos^2 x)^m} \sin x\, dx$

convert to $(1-\cos^2 x)^m$. Use

$\int \sin^{2m} x\, dx = \int (\sin^2 x)^m dx$

1
$1 \quad 1$
$1 \quad 2 \quad 1$
$1 \quad 3 \quad 3 \quad 1$

$= \int \frac{1}{2}[1-\cos(2x)]^m dx$

$\boxed{E}$ $\int \sin^6 x\, dx = \int \frac{1}{2}[1-\cos(2x)]^3 dx = \frac{1}{2}\int [1^3 - 3\cos(2x) + 3\cos^2(2x) - \cos^3(2x)] dx$

$3\left[\frac{1}{2}[1+\cos(4x)]\right]$

$-(1-\sin^2(2x))\cos(2x)$

$(\sin^2 x)^3$

$$\int \sin(\overset{A}{3x}) \cos(\overset{B}{5x})\, dx$$

$$= \int \frac{1}{2} \left[\sin(-2x) + \sin(8x) \right] dx$$

we cool.

α

(15) $$\int \frac{\cos^5 \alpha}{\sqrt{\sin \alpha}}\, d\alpha$$

Scratch:

$\cos^4 \alpha \cos \alpha$

$(1 - \sin^2 \alpha)^2 \cos \alpha$

$1 - 2\sin^2 \alpha + (\sin^2 \alpha)^2$

$= 1 - 2\sin^2 \alpha + \sin^4 \alpha$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^3 = a^3 - 3a^2 b + 3ab^2 - b^3$

$$= \int \sin^{-\frac{1}{2}} \alpha \,(1 - \sin^2 \alpha)^2 \cos \alpha \, d\alpha$$

$$= \int \sin^{-\frac{1}{2}} \alpha \,(1 - 2\sin^2 \alpha + \sin^4 \alpha) \cos \alpha \, d\alpha$$

$$\left.\begin{aligned} &= \int \sin^{-\frac{1}{2}} \alpha \cos \alpha \, d\alpha \\ &\quad - 2\int \sin^{\frac{3}{2}} \alpha \cos \alpha \, d\alpha \\ &\quad + \int \sin^{\frac{7}{2}} \alpha \cos \alpha \, d\alpha \end{aligned}\right\} \text{cake.}$$

$$\int x\cos^2 x\,dx$$

$u = x \Rightarrow du = dx$

$dv = \cos^2 x\,dx \Rightarrow v =$ takes work:

$= \frac{1}{2}[1 + \cos(2x)]dx$

$\int \cos(2x) = \frac{1}{2}\int \cos(2x)(2dx)$

$\Rightarrow v = \frac{1}{2}x + \frac{1}{2}\left(\frac{1}{2}\sin(2x)\right)$

So, $uv - \int v\,du = (x)\left(\frac{1}{2}x + \frac{1}{4}\sin(2x)\right) - \int\left(\frac{1}{2}x + \frac{1}{4}\sin(2x)\right)dx$

$= \frac{1}{2}x^2 + \frac{1}{4}x\sin(2x) - \left[\frac{1}{2}\cdot\frac{1}{2}x^2 - \frac{1}{4}\cdot\frac{1}{2}\cos(2x)\right] + C^*$ STOP HERE ON TEST

$= \frac{1}{2}x^2 + \frac{1}{4}x\sin(2x) - \frac{1}{4}x^2 + \frac{1}{8}\cos(2x) + C^*$

$= \frac{1}{4}x^2 + \frac{1}{4}x\underbrace{\sin(2x)}_{2\sin x\cos x} + \frac{1}{8}\underbrace{\cos(2x)}_{(2\cos^2 x - 1)} + C^*$

$\frac{1}{2}x\cos(x)\sin(x) + \frac{1}{4}x^2 + \frac{1}{4}\cos(x)^2$

Get all done, you differ by something like "$\frac{1}{8}$"

$\int x \cdot \cos(x)^2 \, dx$

$$x \left(\frac{1}{2} \cos(x) \sin(x) + \frac{1}{2} x \right) + \frac{1}{4} \cos(x)^2 - \frac{1}{4} x^2$$

$expand(\%)$

$$\boxed{\frac{1}{2} x \cos(x) \sin(x) + \frac{1}{4} x^2 + \frac{1}{4} \cos(x)^2}$$

$\% - \left(\frac{1}{4} x^2 + \frac{1}{4} x \cdot \sin(2x) + \frac{1}{8} \cos(2x) \right)$

Maple sez. I say differ'nt.

$$\frac{1}{2} x \cos(x) \sin(x) + \frac{1}{4} \cos(x)^2 - \frac{1}{4} x \sin(2x) - \frac{1}{8} \cos(2x)$$

$expand(\%)$

$$\frac{1}{8}$$

$simplify(\%)$

$$x \cos(x) \sin(x) + \frac{1}{2} \cos(x)^2 - \frac{1}{8}$$

$$\int \cot^5 x \sin^4 x \, dx$$
$$= \int \frac{\cos^5 x}{\sin^5 x} \sin^4 x \, dx$$
$$= \int \frac{\cos^5 x}{\sin x} \, dx$$

Powers of tangent & secant

$$\int \tan^5 x \sec^2 x \, dx$$
$$\int u^5 \, du$$
$$\int \sec^{27} x \sec x \tan x \, dx$$
$$= \int u^{27} du$$

$$\sec^2 x = \tan^2 x + 1$$

Even power of secant.

$\int \tan^n x \sec^{2m} x \, dx$, save a $\sec^2 x \, dx$

Do $\int \tan^k x \sec^2 x \, dx$

odd power of tangent

$\int \tan^{2n+1} x \sec^m x \, dx$, save a "$\sec x \tan x$" & write $\tan^{2n} x$ as powers of secant.

$$\int (\sec^k x)(\sec x \tan x) dx$$

See E7, E8 for others.

$\int \sec^3 x \, dx$ $\quad \int \tan^3 x \, dx$

other tools : $\int \tan x \, dx = -\ln|\cos x| + C$

$$-\int \frac{-\sin x}{\cos x} dx$$

$$\int \sec x \, dx = \ln|\tan x + \sec x| + C$$

$= -\ln|\cos x| + C$

$= \ln(|\cos x|^{-1}) + C = \boxed{\ln|\sec x| + C}$