30 $\displaystyle\int_0^1 \frac{r^3}{\sqrt{4+r^2}}\,dr$

$u = r^2 \Longrightarrow du = 2r\,dr$

$dv = \dfrac{r}{\sqrt{4+r^2}}\,dr$

$\Longrightarrow v = \sqrt{4+r^2}$

$\int u\,dv = uv - \int v\,du$

So, $\displaystyle\int_0^1 \frac{r^3}{\sqrt{4+r^2}}\,dr$

$= (r^2)(4+r^2)^{\frac{1}{2}}\Big]_0^1 - \displaystyle\int_0^1 (\sqrt{4+r^2})(2r\,dr)$

$= (1^2)(\sqrt{4+1^2}) - [0^2(\ldots)]$

$- \frac{2}{3}\left[5^{3/2} - 4^{3/2}\right]$

$= \sqrt{5} - \frac{2}{3}\left[5\sqrt{5} - 8\right]$

$5^{\frac{3}{2}} = 5^{1+\frac{1}{2}} = 5^1 5^{\frac{1}{2}} = 5\sqrt{5}$

$= \sqrt{5} - \frac{10}{3}\sqrt{5} + \frac{16}{3}$

$= -\frac{7}{3}\sqrt{5} + \frac{16}{3}$

$\displaystyle\frac{1}{2}\int \frac{2r}{\sqrt{4+r^2}}\,dr$

$= \displaystyle\frac{1}{2}\int u^{-\frac{1}{2}}\,du$

$= \dfrac{1}{2}\,\dfrac{u^{\frac{1}{2}}}{\frac{1}{2}} + C$

$= u^{\frac{1}{2}} + C$

Let $t = 4+r^2$

$\Longrightarrow dt = 2r\,dr$

$r = 0 \Longrightarrow t = 4$

$r = 1 \Longrightarrow t = 5$

$\displaystyle\int_4^5 t^{\frac{1}{2}}\,dt$

$= \frac{2}{3}t^{\frac{3}{2}}\Big]_4^5$

$= \frac{2}{3}\left[5^{\frac{3}{2}} - 4^{\frac{3}{2}}\right]$

$$\frac{d}{dx}\left[3^{10x}\right] = \frac{d}{dx}\left[\left(e^{\ln 3}\right)^{10x}\right]$$

$$= \frac{d}{dx}\left[e^{(\ln 3)(10)x}\right] = e^{(\ln 3)(10)x} \cdot \frac{d}{dx}\left[(\ln 3)(10)x\right]$$

$$= \left(e^{\ln 3}\right)^{10x} \cdot (\ln 3)(10)$$

$$(10 \ln 3)\left(3^{10x}\right)$$

$$\frac{d}{dx}\left[3^{x}\right] = (\ln 3)\left(3^{x}\right)$$

8.2 TRIGONOMETRIC INTEGRALS

Cheat Sheet Stuff:

Pythagorean Trig Identities for sine, cosine, tangent, and their evil twins

Sum, Difference and Half-Angle formulas for sine and cosine.

8.2 #s 6, 9, 12, 18, 27, 28, 37, 38, 44, 55, 62

$$\sin^2 x = \frac{1}{2}\left[1 - \cos(2x)\right]$$

$$\cos^2 x = \frac{1}{2}\left[1 + \cos(2x)\right]$$

$$\sin A \cos B = \frac{1}{2}\left[\sin(A-B) + \sin(A+B)\right]$$

$$\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\cos A \cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \sin x \cos x \, dx = \frac{1}{2}\sin^2 x + C$$

$u = \sin x$

$du = \cos x \, dx$

$$\int u \, du = \frac{1}{2}u^2 + C$$

$u = \cos x$

$du = -\sin x \, dx$, so

$$-\int \cos x \, (-\sin x \, dx)$$

$$= -\frac{1}{2}\cos^2 x + C$$

!?

$$\int \sin^5 x \cos x \, dx$$

$u = \sin x$

$du = \cos x \, dx$

$$\int u^5 du = \frac{1}{6}\sin^6 x + C$$

$$= -\frac{1}{2}(1 - \sin^2 x) + C$$

$$= -\frac{1}{2} + \frac{1}{2}\sin^2 x + C$$

The $-\frac{1}{2}$ can be absorbed into the constant of integration. So, they are the same. :)

$$\int \sin^5 x \cos^3 x \, dx \qquad u^n$$

$$= \int \sin^5 x \, (1-\sin^2 x) \cos x \, dx$$

$$= \int \sin^5 x \cos x - \int \sin^7 x \cos x$$

$$= \frac{\sin^6 x}{6} - \frac{\sin^8}{8} + C$$

---

causes heartburn

$\int \sin^5 x \, dx$ is hard, w/o a $du = \cos x \, dx$ to help.

$\int \sin^4 x \, dx$ see Example 4.

$$\int \underbrace{\sin^4 x} \cdot \sin x \, dx$$

$$= \int (\sin^2 x)^2 \sin x \, dx$$

$$= \int (1-\cos^2 x)^2 \sin x \, dx$$

$$= \int (1 - 2\cos x + \cos^2 x) \sin x \, dx$$

$$= \int \sin x \, dx - 2\int \cos x \sin x \, dx + \int \cos^2 x \sin x \, dx$$

$\int x \cdot \cos(x)^2 \, dx$

$$x\left(\frac{1}{2}\cos(x)\sin(x)+\frac{1}{2}x\right)+\frac{1}{4}\cos(x)^2-\frac{1}{4}x^2$$

$expand(\%)$

$$\frac{1}{2}x\cos(x)\sin(x)+\frac{1}{4}x^2+\frac{1}{4}\cos(x)^2$$

$\% - \left(\frac{1}{4}x^2+\frac{1}{4}x\cdot\sin(2x)+\frac{1}{8}\cos(2x)\right)$

$$\frac{1}{2}x\cos(x)\sin(x)+\frac{1}{4}\cos(x)^2-\frac{1}{4}x\sin(2x)-\frac{1}{8}\cos(2x)$$

$expand(\%)$

$$\frac{1}{8}$$

$simplify(\%)$

$$x\cos(x)\sin(x)+\frac{1}{2}\cos(x)^2-\frac{1}{8}$$