§8.1 Integration by Parts

$$(fg)' = f'g + fg'$$

$$fg = \int (fg)' = \int f'g + \int fg'$$

$$\int fg' = fg - \int f'g = fg - \int gf'$$

$$\int u\,dv = uv - \int v\,du$$

u = something whose derivative is easy

dv = " that you can integrate

$$\int xe^x dx =$$

$$\left.\begin{array}{l} u = x \Rightarrow du = dx \\ dv = e^x dx \Rightarrow v = e^x \end{array}\right\} \Rightarrow$$

$$uv - \int v\,du = xe^x - \int e^x dx = \boxed{xe^x - e^x + C}$$

Check $\frac{d}{dx}\left[xe^x - e^x + C\right] = e^x + xe^x - e^x = xe^x$ ✓

3–32 Evaluate the integral.

$\boxed{3.}$ $\int x \cos 5x \, dx$

$$\int x^{16} e^x dx = x^{16} e^x - 16 \int x^{15} e^x dx$$

$u = x^{16}$, $du = 16x^{15}$
$dv = e^x$, $v = e^x$

$$= x^{16} e^x - 16 \left[x^{15} e^x - 15 \int x^{14} e^x dx \right]$$

$$= x^{16} e^x - 16 x^{15} e^x + 16 \cdot 15 x^{14} e^x - 16 \cdot 15 \cdot 14 x^{13} e^x \ldots$$

But you c'd git there

10. $\int \sin^{-1} x \, dx$

$u = \sin^{-1} x \Rightarrow du = \dfrac{dx}{\sqrt{1-x^2}}$

$dv = dx \Rightarrow v = x$

$$uv - \int v \, du = x \sin^{-1} x - \int \frac{x \, dx}{\sqrt{1-x^2}} \qquad (*)$$

Let $t = 1 - x^2 \Rightarrow dt = -2x \, dx$

$$\frac{dt}{-2x} = dx$$

$$\text{So, } \int \frac{x \, dx}{\sqrt{1-x^2}} = \int \frac{x \cdot \frac{dt}{-2x}}{\sqrt{t}} = -\frac{1}{2} \int \frac{dt}{\sqrt{t}} = -\frac{1}{2} \int t^{-\frac{1}{2}} dt$$

$$= -\frac{1}{2} \cdot \frac{2}{1} t^{\frac{1}{2}} + C_1 = -t^{\frac{1}{2}} + C_1 = -\sqrt{1-x^2} + C_1$$

So, $(*)$ is now

$$x \sin^{-1} x - \left[-\sqrt{1-x^2} + C_1 \right]$$

$$= x \sin^{-1} x + \sqrt{1-x^2} + C, \text{ where } C = -C_1$$

18. $\int e^{-\theta}\cos 2\theta \, d\theta = I$

$u = e^{-\theta} \Rightarrow du = -e^{-\theta}\, d\theta$

$dv = \cos(2\theta)\, d\theta$

$\Rightarrow v = \frac{1}{2}\sin(2\theta)$

$\Rightarrow uv - \int v\, du = \frac{1}{2}e^{-\theta}\sin(2\theta) + \frac{1}{2}\int e^{-\theta}\sin(2\theta)\, d\theta$

$= \frac{1}{2}e^{-\theta}\sin(2\theta) + \frac{1}{2}I_1$

$= ?$

Let $I_1 = \int e^{-\theta}\sin(2\theta)\, d\theta$

Let $u = e^{-\theta} \Rightarrow du = -e^{-\theta}\, d\theta$

$dv = \sin(2\theta)\, d\theta \Rightarrow v = -\frac{1}{2}\cos(2\theta)$

$\Rightarrow I_1 = (e^{-\theta})\left(-\frac{1}{2}\cos(2\theta)\right) - \int\left(-\frac{1}{2}\cos(2\theta)\right)(-e^{-\theta}\, d\theta)$

$= -\frac{1}{2}e^{-\theta}\cos(2\theta) - \frac{1}{2}\int e^{-\theta}\cos(2\theta)\, d\theta = I_1$

$I = \frac{1}{2}e^{-\theta}\sin 2\theta + \frac{1}{2}I_1$

$= \frac{1}{2}e^{-\theta}\sin 2\theta - \frac{1}{4}e^{-\theta}\cos(2\theta) - \frac{1}{4}I$

$= \frac{1}{2}e^{-\theta}\sin(2\theta) - \frac{1}{4}e^{-\theta}\cos(2\theta) - \frac{1}{4}I$

$\frac{5}{4}I = \frac{1}{2}e^{-\theta}\sin(2\theta) - \frac{1}{4}e^{-\theta}\cos(2\theta)$

$I = \frac{4}{5}\left[\frac{1}{2}e^{-\theta}\sin(2\theta) - \frac{1}{4}e^{-\theta}\cos(2\theta\right.$

11. $\int \arctan 4t\, dt$