Testing w/ Smartboard

$$\lim_{x\to\infty} \frac{e^x}{x^3} \overset{H}{=} \lim_{x\to\infty} \frac{e^x}{3x^2} \overset{H}{=} \lim_{x\to\infty} \frac{e^x}{6x} \overset{H}{=} \lim_{x\to\infty} \frac{e^x}{6} = \infty$$

$$\lim_{x\to 0^+} \frac{\ln x}{x} = -\infty \qquad \frac{-\infty}{0}$$

$$0 < x \leq 1, \quad \frac{\ln x}{x} \leq \ln x \xrightarrow{x\to 0^+} -\infty$$

Arguing that the x in the denominator just makes it approach $-\infty$ more quickly than $\ln x$ by itself!

7.6 I run-thru

#18

(a) $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

Goal: $\frac{d}{dx}\cos^{-1}x = -\frac{1}{\sqrt{1-x^2}}$

$-1 < x < 1$

$\boxed{\text{Pf}}$ Sorta ok sorta cheatin'

$\frac{d}{dx}$[top line]:

$$\frac{1}{\sqrt{1-x^2}} + -\frac{1}{\sqrt{1-x^2}} = 0, \text{ so } \sin^{-1}x + \cos^{-1}x = \text{constant}$$

So, let $x=1$ in $\sin^{-1}x + \cos^{-1}x$

$$= \sin^{-1}1 + \cos^{-1}1 = \frac{\pi}{2} + 0 = \frac{\pi}{2}$$

Let $a = \sin^{-1}x$, $b = \cos^{-1}x$

So, $a+b = 90^\circ = \frac{\pi}{2}$ radians.

Clobbers (a)

(b) $\frac{d}{dx}\left[\cos^{-1}x\right] = -\frac{1}{\sqrt{1-x^2}}$, using part (a) result.

3 hw pts for it.

7.8 stuff

#57 $\lim_{x\to\infty}\left(1+\frac{3}{x}+\frac{5}{x^2}\right)^x$

Let $y=\left(1+\frac{3}{x}+\frac{5}{x^2}\right)^x \Longrightarrow$ 1^∞ $\ln\left(\left(1+\frac{3}{x}+\frac{5}{x^2}\right)^x\right)$

$\lim_{x\to\infty}\ln y = \lim_{x\to\infty} x\ln\left(1+\frac{3}{x}+\frac{5}{x^2}\right)$ → 0 as $x\to\infty$ $\infty\cdot 0$

x → ∞ as $x\to\infty$

$$=\lim_{x\to\infty}\frac{\ln\left(1+\frac{3}{x}+\frac{5}{x^2}\right)}{\frac{1}{x}}$$

Teacher doesn't like his notation choices, here.

$$=\lim_{x\to\infty}\frac{\frac{-\frac{3}{x^2}-\frac{10}{x^3}}{1+\frac{3}{x}+\frac{5}{x^2}}}{-\frac{1}{x^2}}$$

A little better, now

$$=\lim_{x\to\infty}\frac{\frac{1}{x^2}\left(-3-\frac{10}{x}\right)\left(\frac{1}{1+\frac{3}{x}+\frac{5}{x^2}}\right)}{-\frac{1}{x^2}} = \frac{-3}{-1}=3$$

$\frac{10}{x}\to 0$, $\frac{3}{x}\to 0$, $\frac{5}{x^2}\to 0$

is $\lim_{x\to\infty}\ln y$. We want $\lim_{x\to\infty} y = e^3$ is the final answer

$\lim_{x\to\infty}\ln y = 3$

$\lim_{x\to\infty} e^{\ln y} = e^3 = \lim_{x\to\infty} y$

#51. $\lim_{x\to\infty}(x-\ln x)$ $\quad \infty-\infty$

$= \lim_{x\to\infty}\left(x\left(1-\frac{\ln x}{x}\right)\right)$ $\quad \infty\cdot 1$

$\frac{\ln x}{x} \to 0$ as $x\to\infty$

$x \to \infty$, $\left(1-\frac{\ln x}{x}\right) \to 1$

$e^x > x^n > \ln x$ for large x.

$= \infty$

#59 $\lim_{x\to\infty}\left(x^{\frac{1}{x}}\right)$ $\quad \infty^0$

$y = x^{\frac{1}{x}} \Rightarrow$

$\ln y = \frac{1}{x}\ln x = \frac{\ln x}{x} \xrightarrow{x\to\infty} 0$

$e \quad e \quad e \quad e$

$\Rightarrow y = x^{\frac{1}{x}} \xrightarrow{x\to\infty} e^0 = 1.$

#92 f'' is cnts $\Rightarrow$

$f''(x) = \lim_{h\to 0}\frac{f(x+h)-2f(x)+f(x-h)}{h^2}$

#93 $\boxed{\lim_{x\to\infty}\frac{e^x}{x^n} = \infty \quad P(n)}$ $\quad f(x)^{g(x)}$

#94 $\lim_{x\to\infty}\frac{\ln x}{x^p} = 0$, $p>0$

Proof by Induction.

Let $S = \{n\in\mathbb{N} \mid P(n) \text{ holds}\}$

Note that $\frac{e^x}{x} \xrightarrow[\text{H}]{x\to\infty} \frac{e^x}{1} \xrightarrow{x\to\infty} \infty$

This shows $1\in S \neq \emptyset$. Let $1\le k\in S$.

Then $\frac{e^x}{x^k} \xrightarrow{x\to\infty} \infty$

Consider $\lim_{x\to\infty}\frac{e^x}{x^{k+1}} \overset{\text{H}}{=} \lim_{x\to\infty}\frac{e^x}{(k+1)x^k} = \frac{1}{k+1}\lim_{x\to\infty}\frac{e^x}{x^k} = \frac{1}{k+1}\cdot\infty = \infty,$

since $k\in S$ means the $P(k)$ holds.

FINISH ALL HOMEWORKS

CHEAT SHEET PROPOSALS

Test so far from Chapter Review

Couple T-F

OMIT #48, 60, 79-84, 106-108, 118 to END