7.6 INVERSE TRIGONOMETRIC FUNCTIONS

Part II 7.6 II #s 51, 58, 60, 64, 70

23. $y = \tan^{-1}\sqrt{x} \implies y' = \dfrac{1}{1+(\sqrt{x})^2} \cdot \dfrac{1}{2}x^{-\frac{1}{2}}$

$\sqrt{x} = x^{\frac{1}{2}}$

$= \dfrac{1}{1+x} \cdot \dfrac{1}{2\sqrt{x}}$

$= \dfrac{1}{2\sqrt{x}\,(1+x)}$

$\sqrt{\ }$ $(x \geq 0)$ $(x > 0)$

$\dfrac{\ }{\sqrt{x}\,(\ldots)}$

60. $\displaystyle\int \frac{\tan^{-1}x}{1+x^2}\,dx = \int u\,du$

$u = \tan^{-1}x$

$\implies du = \dfrac{1}{1+x^2}\,dx$

$= \frac{1}{2}u^2 + C$

$= \frac{1}{2}(\tan^{-1}x)^2 + C$

63. $\displaystyle\int \frac{1+x}{1+x^2}\,dx = \int \frac{dx}{1+x^2} + \frac{1}{2}\int \frac{2x\,dx}{1+x^2}$

$u = 1+x^2$

$du = 2x\,dx$

$= \tan^{-1}x + \frac{1}{2}\ln(1+x^2) + C$

15–16 Graph the given functions on the same screen. How are these graphs related?

15. $y = \sin x,\ -\pi/2 \leqslant x \leqslant \pi/2;\quad y = \sin^{-1}x;\quad y = x$

(41) Find $f'(x)$ check by graphing f & f'.

$$f(x) = \sqrt{1-x^2}\ \arcsin x$$

$$\frac{-\cancel{2}x}{\cancel{2}\sqrt{1-x^2}}\arcsin x + \cancel{\sqrt{1-x^2}} \cdot \frac{1}{\cancel{\sqrt{1-x^2}}}$$

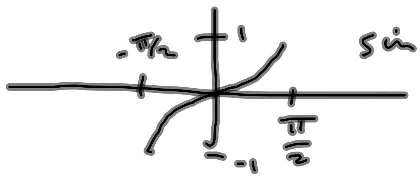
$$= 1 - \frac{x \arcsin x}{\sqrt{1-x^2}}$$

$\boxed{10}$ $y = \csc^{-1}x \ (|x| \geqslant 1) \iff \csc y = x$ and $y \in (0, \pi/2] \cup (\pi, 3\pi/2]$

$y = \sec^{-1}x \ (|x| \geqslant 1) \iff \sec y = x$ and $y \in [0, \pi/2) \cup [\pi, 3\pi/2)$

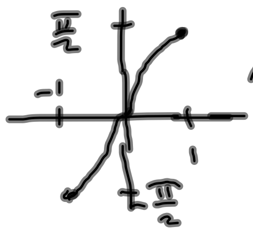
$y = \cot^{-1}x \ (x \in \mathbb{R}) \iff \cot y = x$ and $y \in (0, \pi)$

43–46 Find the limit.



sin x

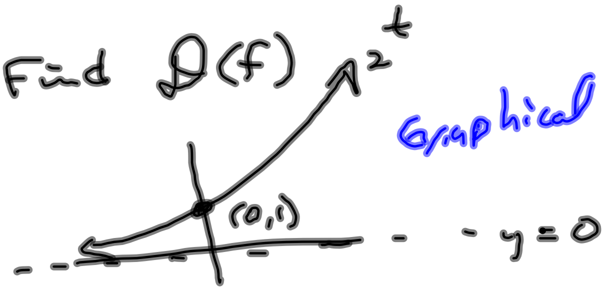
43. $\lim_{x \to -1^+} \sin^{-1}x = -\frac{\pi}{2}$



Arcsin x

7.2 #s 16, 54, 74, 84

16 $\sqrt{1-2^t} = f(t)$ Find $\mathcal{D}(f)$



Graphical

Nee $\underline{1-2^t \geq 0}$

$2^t \leq 1$

$\ln(2^t) \leq \ln 1$

$\log_2(2^t) \leq \log_2 1$

$t \leq 0$ Analytical

$\mathcal{D} = \{t \mid t \leq 0\}$

$t \ln 2 \leq \ln 1$

$(\ln 2) t \leq 0$

$t \leq 0$

54 $y + y' = y''$

$y'' - y' - y = 0$

$D^2 y - Dy - y = 0$

$(D^2 - D - 1) y = 0$

$D^2 - D - 1 = 0$

$y = e^{\lambda t} \Rightarrow y' = \lambda e^{\lambda t} \Rightarrow y'' = \lambda^2 e^{\lambda t}$

$\lambda^2 e^{\lambda t} - \lambda e^{\lambda t} - e^{\lambda t} = 0$

$e^{\lambda t}(\lambda^2 - \lambda - 1) = 0$

$\lambda^2 - \lambda - 1 = 0$

$\lambda^2 - \lambda + \left(\frac{1}{2}\right)^2 = 1 + \frac{1}{4} = \frac{5}{4}$

$\frac{1}{2} \rightsquigarrow \left(\frac{1}{2}\right)^2$

$\sqrt{\left(\lambda - \frac{1}{2}\right)^2} = \sqrt{\frac{5}{4}}$

$\left|\lambda - \frac{1}{2}\right| = \frac{\sqrt{5}}{2}$

$\lambda - \frac{1}{2} = \pm \frac{\sqrt{5}}{2}$

$\lambda = \frac{1 \pm \sqrt{5}}{2}$