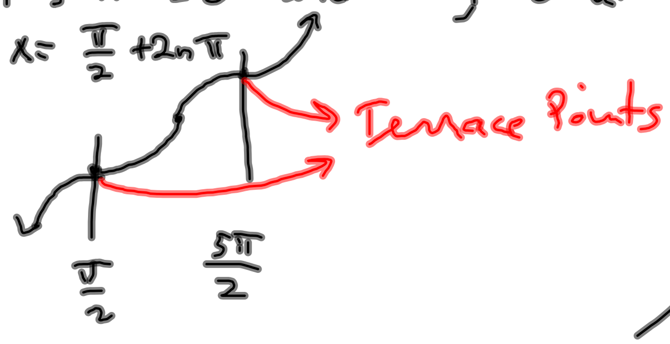
7.1

$f(x) = x + \cos x$

$f'(x) = 1 - \sin x \geq 0$ and only $= 0$ at $x = \frac{\pi}{2} + 2n\pi$



Want $f^{-1}(1)$ SET $f(x) = 1$

$x + \cos x = 1$

NOTE $0 + \cos(0) = 1$? Good Guess!

$0 + 1 = 1$!

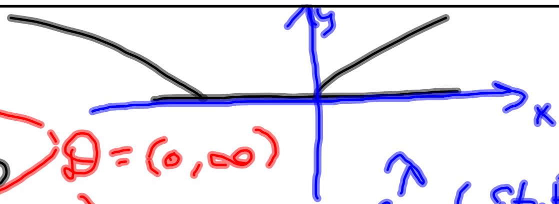
$f(x) = 1$ when $x = 0$

$(0, 1) = (x, f(x))$

$(1, 0) = (f^{-1}(x), x)$

$x = 1 \Rightarrow \boxed{f^{-1}(1) = 0}$

7.1 #30



$f(x)=\sqrt{x^2+2x}$ $\quad x>0 \quad \mathcal{D}=(0,\infty)$

$\mathcal{R}=(0,\infty)$

Good Stuff.

$y=\sqrt{x^2+2x}$

$\sqrt{y^2+2y}=x$ Solve for y. Text switches variables in last step.

$y^2+2y=x^2$

$y^2+2y+1^2=x^2+1$

$\frac{2}{2}=1 \rightsquigarrow 1^2$

$(y+1)^2=x^2+1$

$y+1=\pm\sqrt{x^2+1}$

$y=-1\pm\sqrt{x^2+1}=f^{-1}(x)$, almost.

$\boxed{\begin{array}{c}\mathcal{D}(f^{-1})=\mathcal{R}(f)\\ \mathcal{R}(f^{-1})=\mathcal{D}(f)\end{array}}$

Need

$-1+\sqrt{x^2+1}$.

Throw out the "−" part.

36 Assume $f(x_1) = f(x_2)$

Then $\frac{1}{x_1-1} = \frac{1}{x_2-1}$

$\Rightarrow x_2 - 1 = x_1 - 1$

$\Rightarrow x_2 = x_1$

$\Rightarrow$ 1-to-1.

Same as what book does.

(b) T7 for $(f^{-1})'(a)$

$$(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}$$

$f(x) = \frac{1}{x-1} = (x-1)^{-1} \Rightarrow f'(x) = \boxed{-(x-1)^{-2}}$

$\frac{1}{x-1} = a$ to find $f^{-1}(a)$, solve $f(x) = a$

$1 = a(x-1)$

$\frac{1}{a} = x - 1$

$\frac{1}{a} + 1 = x$

$$\frac{1}{-\left(\frac{1}{a}+1-1\right)^{-2}} = \frac{1}{-\left(\frac{1}{a}\right)^{-2}} = -\left(\frac{1}{a}\right)^{2}$$

$a = 2$: $-\left(\frac{1}{2}\right)^2 = -\frac{1}{4}$

7.6 INVERSE TRIGONOMETRIC FUNCTIONS

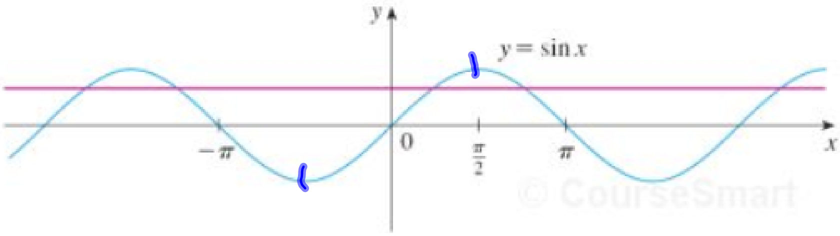
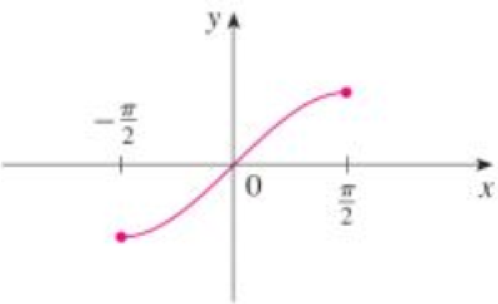


FIGURE 1



$D = [-\frac{\pi}{2}, \frac{\pi}{2}]$

$R = [-1, 1]$

FIGURE 2 $y = \sin x, -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

1 $$\boxed{\sin^{-1}x = y \iff \sin y = x \quad \text{and} \quad -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}}$$

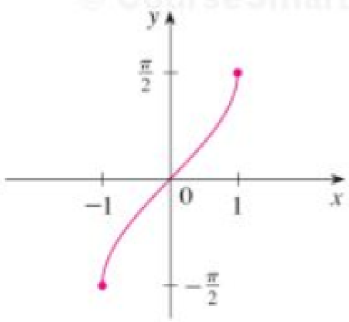


FIGURE 4
$y = \sin^{-1}x = \arcsin x$

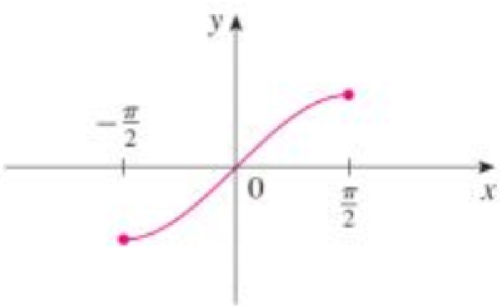


FIGURE 2 $y = \sin x, -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

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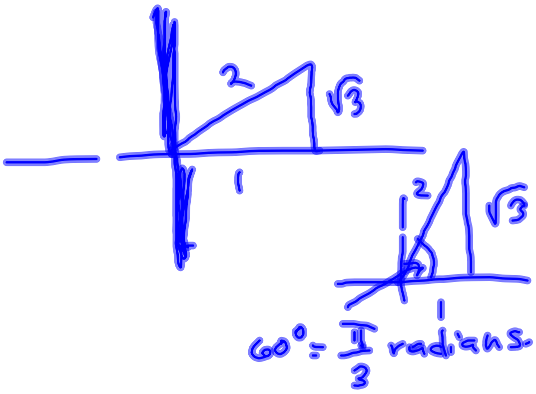
1–10 Find the exact value of each expression.

(See also, Example 1)

1. (a) $\sin^{-1}(\sqrt{3}/2) = \frac{\pi}{3}$



4. (a) $\cot^{-1}(-\sqrt{3})$ (b) $\arccos(-\frac{1}{2}) = \frac{2}{3}\pi$

$D(\cot^{-1}) = ?$

$(-\infty, \infty)$?

Trig Review Back of Book. For $D(\cot^{-1})$

$\pi - \frac{\pi}{3} = \frac{2}{3}\pi$

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2

$$\sin^{-1}(\sin x) = x \quad \text{for } -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

$$\sin(\sin^{-1} x) = x \quad \text{for } -1 \leq x \leq 1$$

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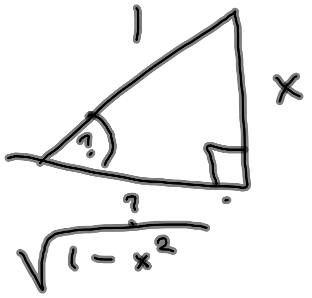
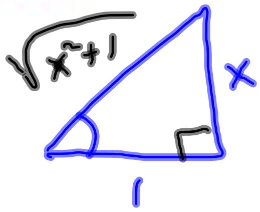
12–14 Simplify the expression.

12. $\tan(\sin^{-1}x) = \dfrac{x}{\sqrt{1-x^2}}$



13. $\boxed{13.}$ $\sin(\tan^{-1}x) = \dfrac{x}{\sqrt{x^2+1}}$



15–16 Graph the given functions on the same screen. How are these graphs related?

15. $y = \sin x,\ -\pi/2 \leqslant x \leqslant \pi/2;\quad y = \sin^{-1}x;\quad y = x$

3

$$\boxed{\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}} \qquad -1 < x < 1}$$

PROOF: (This again relates to Brent's 7.1 #40, and comments from 7.5 on Friday.)

44

Let $y = \sin^{-1}x$. Then $\sin y = x$ and $-\pi/2 \leqslant y \leqslant \pi/2$. Differentiating $\sin y = x$ implicitly with respect to x, we obtain

$$\cos y \frac{dy}{dx} = 1$$

and

$$\frac{dy}{dx} = \frac{1}{\cos y}$$

Now $\cos y \geqslant 0$ since $-\pi/2 \leqslant y \leqslant \pi/2$, so

$$\cos y = \sqrt{1 - \sin^2 y} = \sqrt{1 - x^2}$$

Therefore

$$\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-x^2}}$$

See Example 2 for an application. *

4 $$\cos^{-1}x = y \iff \cos y = x \quad \text{and} \quad 0 \leq y \leq \pi$$

5 $$\cos^{-1}(\cos x) = x \quad \text{for } 0 \leq x \leq \pi$$
$$\cos(\cos^{-1}x) = x \quad \text{for } -1 \leq x \leq 1$$

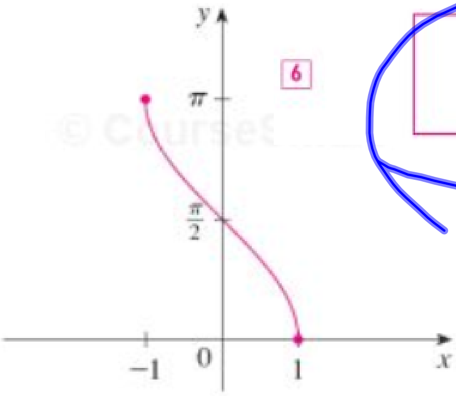


FIGURE 7
$y = \cos^{-1}x = \arccos x$

6 $$\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}} \quad -1 < x < 1$$

7 $$\tan^{-1}x = y \iff \tan y = x \quad \text{and} \quad -\frac{\pi}{2} < y < \frac{\pi}{2}$$

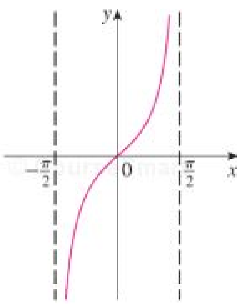


FIGURE 8
$y = \tan x, -\frac{\pi}{2} < x < \frac{\pi}{2}$

8 $$\lim_{x\to\infty} \tan^{-1}x = \frac{\pi}{2} \qquad \lim_{x\to-\infty} \tan^{-1}x = -\frac{\pi}{2}$$

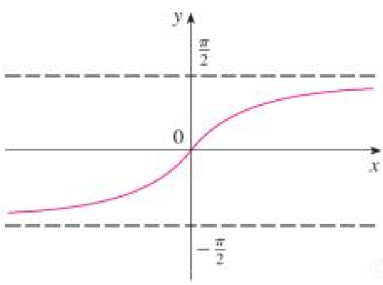


FIGURE 10
$y = \tan^{-1}x = \arctan x$

Aug 31-9:56 AM

9 $$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$$

Proof: (Leaving proofs of other derivatives as exercises.)

$y = \tan^{-1}x$. Then $\tan y = x$. Differentiating this latter equation implicitly with respect to x, we have

$$\sec^2 y \frac{dy}{dx} = 1$$

and so

$$\frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{1+\tan^2 y} = \frac{1}{1+x^2}$$

9 $$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$$

10 $y = \csc^{-1}x \ (|x| \geq 1) \iff \csc y = x$ and $y \in (0, \pi/2] \cup (\pi, 3\pi/2]$

$y = \sec^{-1}x \ (|x| \geq 1) \iff \sec y = x$ and $y \in [0, \pi/2) \cup [\pi, 3\pi/2)$

$y = \cot^{-1}x \ (x \in \mathbb{R}) \iff \cot y = x$ and $y \in (0, \pi)$

#s 19, 20, 21 are the remaining proofs of the derivative formulas for the inverse trig functions. See the discussion on page 458 about choice of domain for csc x and sec x. We'll do a little bit with all of them, but mostly arcsine, arccosine and arctangent.

22–35 Find the derivative of the function. Simplify where possible.

23. $y = \tan^{-1}\sqrt{x}$

Stick these on a cheat sheet.

11 TABLE OF DERIVATIVES OF INVERSE TRIGONOMETRIC FUNCTIONS

$$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1 - x^2}} \qquad \frac{d}{dx}(\csc^{-1}x) = -\frac{1}{x\sqrt{x^2 - 1}}$$

$$\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1 - x^2}} \qquad \frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2 - 1}}$$

$$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1 + x^2} \qquad \frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1 + x^2}$$

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43–46 Find the limit.

43. $\lim\limits_{x \to -1^+} \sin^{-1}x$

36–37 Find the derivative of the function. Find the domains of the function and its derivative.

36. $f(x) = \arcsin(e^x)$

41–42 Find $f'(x)$. Check that your answer is reasonable by comparing the graphs of f and f'.

41. $f(x) = \sqrt{1 - x^2} \arcsin x$

7.6 I #s 2, 5, 8, 14, 16, 18, 21, 28, 30, 44 Wed

7.6 II under construction.

For Wednesday, I want teams of 3 to prepare 1 example each (Odd (wo)man goes with team of his/her choice)

#48: Brent, David, Joel
#50: Aaron, Nicholas, Johnathan
#54: Kevin, James, Zachary

Barbara: Take your pick (you're singled out because you're last alphabetically - I'll change this up as semester progresses)

7.6 II #s 51, 58, 60, 64, 70

Aug 31-10:09 AM